PHYSICS FOR INFORMATION SCIENCE - PH3256

UNIT : 1

Electrical Properties of Materials

Classical free electron theory - Expression for electrical Conductivity. Thermal conductivity, expression - Widemann - Franz law - Success and failures - electrons in metals - Particle in a three dimensional box - degenerate states - Fermi - Dirac Statistics - Density of energy states - Electron in periodic potential - Energy bands in solids - tight binding approximation - Electron effective mass - Concept of hole.

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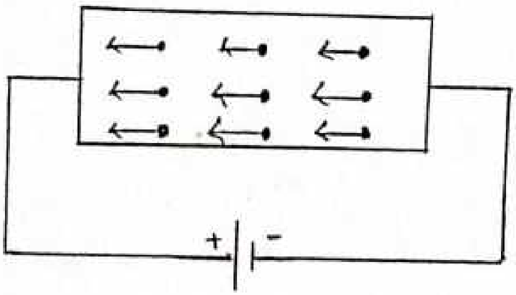
① Give the Assumptions or postulates of Classical free electron theory.



1. A solid metal has nucleus with revolving electrons. The electrons move freely like molecules in a gas.

2. The free electrons move in a uniform potential field due to the ions fixed in the lattice.

3. In the absence of electric field (E = 0), the free electrons move in random directions and collide with each other.

- Since the collisions are elastic

4. when the presence of electric field (E ≠ 0) the free electrons are accelerated in the direction opposite to the direction of applied electric field.

5. Electrons are assumed to be Perfect gas, they obey the laws of classical theory of gases.

6. Classical free electrons in the metal obey Maxwell - Boltzmann Statistics.

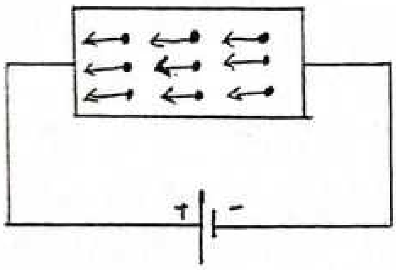
7. Electrons acquire a constant average velocity known as drift velocity.

8. Free electrons can be assigned with mean free path, mean Collision time and average Spee

② Derive the expressions for electrical conductivity and thermal Conductivity metal based on classical free electron theory. State and prove Widemann - Franz law. and Lorentz number

i) Expression for Electrical Conductivity [Based on Drude and Lorentz]

When an electrical field (E) is applied to an electron of charge 'e' of a metallic rod, the electron moves in opposite direction to the applied field with a velocity v_d. This velocity is known as drift velocity.



Force experienced by the electron

$$F = eE \longrightarrow (1)$$

From Newton's Second law of motion

$$F = ma \longrightarrow (2)$$

From eqns (1) and (2)

$$ma = eE$$

$$a = \frac{eE}{m} \longrightarrow (3)$$

From eqn (3) electron should be accelerated continuously due to the applied electric field.

- After each collision the velocity of electron increases untill the next collision takes place.

Average drift velocity of electron = v_d

If τ_c is collision time then acceleration

$$a = \frac{v_d}{\tau} \qquad (\therefore \tau_c = \tau)$$

$$v_d = a\tau \longrightarrow (4)$$

Substituting eqn (3) in eqn (4)

$$v_d = \frac{eE}{m}\tau$$

$$v_d = \left(\frac{e\tau}{m}\right)E \longrightarrow (5)$$

The current density J is related to the drift velocity

$$J = nev_d \longrightarrow (6)$$

Substituting eqn (5) in eqn (6)

$$J = ne\left(\frac{e\tau}{m}\right)E$$

$$\frac{J}{E} = \frac{ne^2\tau}{m} \longrightarrow (7)$$

According to ohm's law, current density J is

$$J = \sigma E \quad , \quad \sigma = J/E \longrightarrow (8)$$

Comparing eqn (7) and (8)

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \longrightarrow (9)$$

The eqn (9) is called electrical conductivity of the metal.

③

ii) Expression for Thermal Conductivity (K) of a Metal

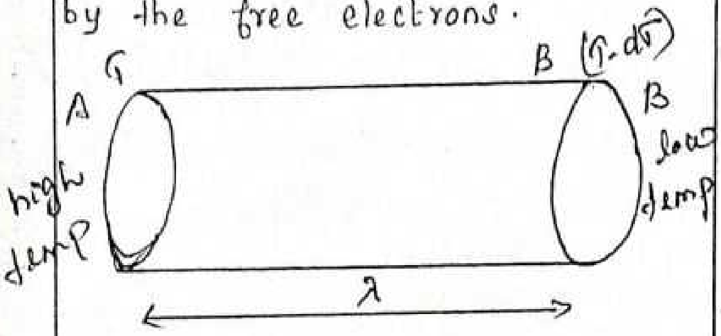
Definition

Thermal Conductivity (K) of a metal is defined as the amount of heat (Q) conducted per unit area (A) per unit time (t) maintained at unit temperature gradient.

Consider two cross-sections A and B of a metal rod seperated by a distance λ.

- A be a high temperature (T)
- B at a low temperature ($T-dT$)

Now heat flows from A to B by the free electrons.



Conduction electron per unit volume $= n$

Average velocity of the electrons $= v$

- Elastic collision takes place

At A average kinetic energy of an electron $= \frac{3}{2} kT \rightarrow (1)$

$$\left(K.E = \frac{1}{2} mv^2 = \frac{3}{2} kT \right)$$

where

k - Boltzmann's Constant

T - Temperature at A.

At B, average K.E of the electron

$$= \frac{3}{2} k \left(T - dT \right) \longrightarrow (2)$$

The excess of K.E carried by the electron from A to B

$$= \frac{3}{2} kT - \frac{3}{2} k (T - dT)$$

$$= \cancel{\frac{3}{2} kT} - \cancel{\frac{3}{2} kT} + \frac{3}{2} k dT$$

$$= \frac{3}{2} k dT \longrightarrow (3)$$

Number of electrons crossing per unit time from A to B $= \frac{1}{6} nv$

$$\longrightarrow (4)$$

The excess of energy carried from A to B per unit area in unit time

$$= \frac{1}{6} nv \times \frac{3}{2} k dT$$

$$= \frac{1}{4} nvk dT \longrightarrow (5)$$

The deficient of energy carried from B to A unit area per unit time

$$= -\frac{1}{4} nvk dT \longrightarrow (6)$$

The net amount of energy transferred from A to B per unit area per unit time

$$Q = \frac{1}{4} nvk dT - \left(-\frac{1}{4} nvk dT \right)$$

$$= \frac{1}{4} nvk dT + \frac{1}{4} nvk dT$$

$$Q = \left(\frac{1}{4} + \frac{1}{4} \right) nvk dT$$

$$Q = \frac{1}{2} n v k \, dT \longrightarrow (7)$$

From the definition of thermal conductivity, $Q = K \, dT/\lambda$

$$\frac{1}{2} n v k \, dT = K \, dT/\lambda \longrightarrow (8)$$

For the metals

Relaxation time = Collision time

$$\tau = \tau_c = \lambda/v$$

$$\tau v = \lambda \longrightarrow (9)$$

Substituting eqn (9) and eqn (8)

$$K = \frac{1}{2} n v k \tau v$$

$$K = \frac{1}{2} n v^2 k \tau \longrightarrow (10)$$

The eqn (10) is the expression for the thermal conductivity. of a metal

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iii) Widemann - Franz law

Law :

The ratio between the thermal conductivity and electrical conductivity of a metal is directly proportional to the absolute temperature of the metal

$$\frac{K}{\sigma} \propto T$$

$$\frac{K}{\sigma} = LT$$

where, L - Proportionality constant, and it is known as Lorentz number.

Derivation

Thermal conductivity of a metal

$$K = \frac{1}{2} n v^2 k \tau \longrightarrow (1)$$

Electrical conductivity of the metal

$$\sigma = \frac{ne^2\tau}{m} \longrightarrow (2)$$

$$\frac{\text{Thermal conductivity}}{\text{Electrical Conductivity}} = \frac{K}{\sigma}$$

$$= \frac{\frac{1}{2} n v^2 k \tau}{\frac{ne^2\tau}{m}}$$

$$= \frac{1}{2} \frac{\cancel{n} v^2 k \cancel{\tau}}{\cancel{n} e^2 \cancel{\tau}} \times m$$

$$\frac{K}{\sigma} = \frac{1}{2} \frac{m v^2 k}{e^2} \longrightarrow (3)$$

The K.E of the electron is given by

$$\frac{1}{2} m v^2 = \frac{3}{2} kT \longrightarrow (4)$$

Substituting the eqn(4) in eqn(3)

$$\frac{K}{\sigma} = \frac{3}{2} \frac{kT \times k}{e^2}$$

$$= \frac{3}{2} \frac{k^2 . T}{e^2}$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{K^2}{e^2}\right)T$$

$$\frac{k}{\sigma} = LT \longrightarrow (5)$$

where $L = \frac{3}{2}\left(\frac{K^2}{e^2}\right)$ is Constant and known as Lorentz number.

$$\boxed{\frac{K}{\sigma} \propto T} \longrightarrow (6)$$

Hence it is verified.

iv) Lorentz Number

The ratio of thermal conductivity to the product of electrical conductivity and absolute temperature of the metal is constant. It is known as Lorentz number and it is given by

$$\boxed{L = K/\sigma T}$$

$$L = \frac{3}{2}\left(\frac{k}{e}\right)^2$$

Boltzmann's constant,

$K = 1.38 \times 10^{-23} JK^{-1}$

charge of an electron,

$e = 1.602 \times 10^{-19}$ Coulomb

$$L = \frac{3}{2}\left(\frac{1.38 \times 10^{-23}}{1.602 \times 10^{-19}}\right)^2$$

$$L = 1.12 \times 10^{-8} \, W \Omega K^{-2}$$

② Write about Success and failures of classical free electron theory.

Success

- It is used to verify ohm's law.
- It is used to explain electrical and thermal conductivities of metals.
- It is used to derive Widemann - Franz law.
- It is used to explain the optical properties of metals.

Failures (or) Drawbacks

- It is a macroscopic theory
- According to classical free electron theory, all the free electrons will absorb energy, but the quantum free electron theory states that only few electrons will absorb energy.
- This theory cannot explain the Compton effect, Photo - electric

effect, Paramagnetism and Ferromagnetism etc

- This theory cannot explain the electrical conductivity of semiconductors and insulators.
- Dual nature of light radiation cannot be explained.
- The theoretical and experimental values of specific heat and electronic specific heat are not matched.
- By classical theory $\frac{K}{\sigma T}$ is constant for all temperature, however it is found that it is not a constant at low temperatures.
- The Lorentz number by classica theory does not have good agreement with the experimental value and theoretical value and it is rectified by quantum theory.

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③ Define Fermi distribution function. How does energy leves and fermi enery vary with temperature and also give its importance.

Fermi Distribution function

Fermi function F(E) represents probability of the electrons occupying in the given energy level at absolute temperature. It is given by

$$\boxed{F(E) = \frac{1}{1+e^{(E-E_f)/kT}}}$$

where,

E - Energy of the level

E_f - Fermi energy level

k - Boltzmann's constant.

T - Absolute Temperature.

The probability value F(E) lies between 0 and 1.

- If F(E) = 1, the energy level is occupied by an electron.
- If F(E) = 0, the energy level is vacant.

- If $F(E) = 0.5$ or $1/2$, then there is a 50% chance for the electron occupying in that energy level.

Effect of temperature on Fermi function

Case (i) Probability of occupation for $E < E_F$ at $T = 0K$

when $T = 0K$ and $E < E_F$

$$F(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$$

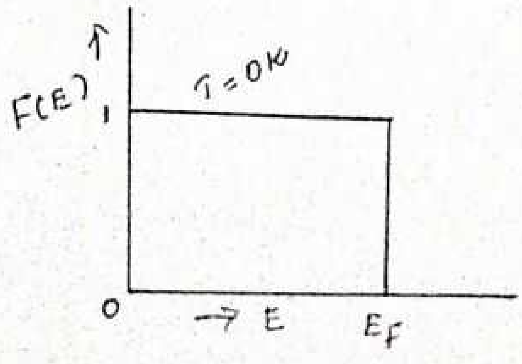
$$F(E) = \frac{1}{1 + e^{(-ve/0)}}$$

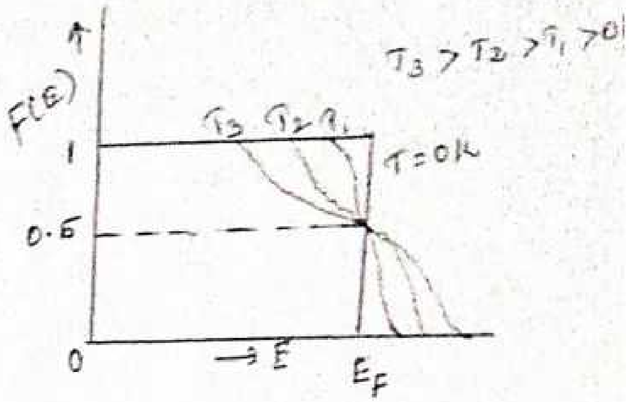
$$F(E) = \frac{1}{1 + e^{-\infty}} \quad : \left[e^{-\infty} = 0\right]$$

$$= \frac{1}{1+0} = \frac{1}{1}$$

$$\boxed{F(E) = 1}$$

At $T = 0K$, there is 100% chance for the electrons to occupy the energy levels below fermi energy level.





Case (ii): Probability of occupation for $E > E_F$ at $T = 0K$

when $T = 0K$, $E > E_F$

$$F(E) = \frac{1}{1 + e^{(+ve/0)}}$$

$$= \frac{1}{1 + e^{\infty}} \quad \left[e^{\infty} = \infty\right]$$

$$= \frac{1}{1 + \infty}$$

$$= \frac{1}{\infty} = 0$$

$$\boxed{F(E) = 0}$$

There is 0% chance for the electrons to occupy the energy level above fermi energy level.

Case (iii) Probability of occupation at ordinary temperature

At ordinary temperature the value of the probability function starts reducing from 1 energy values E slightly less than E_F.

At any temperature other than 0K and $E = E_F$

$$F(E) = \frac{1}{1+e^0}$$

$$F(E) = \frac{1}{1+1} = \frac{1}{2}$$

$$= 0.5 \qquad [e^0 = 1]$$

$$\% \text{ of } F(E) = 0.5 \times 100$$

$$= 50\ \%$$

There is a 50 % chance for the electrons to occupy Fermi energy level.

Case (iv) : At high temperature

when $kT >> E_F$ or $T \rightarrow \infty$, the electrons lose their quantum mechanical character.

Uses of Fermi distribution function

- It gives the Probability of an electron occupancy for a given energy level at a given temperature.
- It is very useful to find the number of free electrons per unit volume at a given temperature.
- It is used to find Fermi energy of the metal.

Importance : It is the reference energy level which separates the filled energy levels and vacant energy levels.

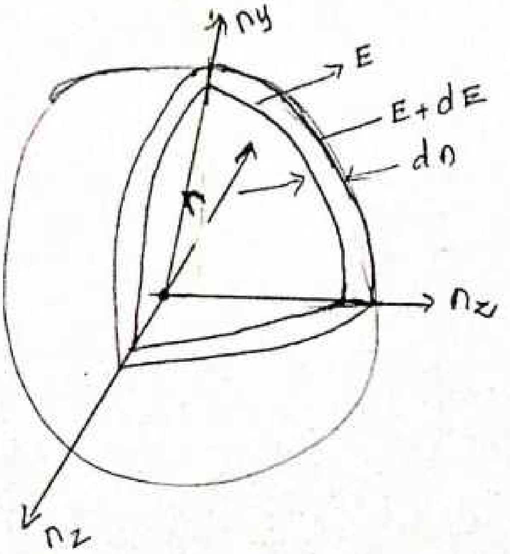
④ obtain an expression for density of states

Definition :

It is defined as the number of available energy states per unit volume in an energy interval E and E+dE

$$Z(E)dE = \frac{\text{Number of energy states in between energy } E \text{ and } E+dE \text{ in a metal piece } N(E)dE}{\text{volume of the metal piece } (V)}$$

Derivation



- Consider a cubical side 'a'
- available states E and E+dE
- n_x, n_y, n_z - Coordinate axis
- radius vector - n
- Origin O
- n_x, n_y, n_z - Coordinates

The number of energy states with in a sphere of radius 'n'

$$= \frac{4}{3}\pi n^3 \text{ (volume of the sphere)} \longrightarrow (1)$$

- $\left(1/8\right)^{th}$ of the spherical volume has to be considered.

The number of available energy states with in one octant of the sphere of radius 'n' corresponding to energy E

$$= 1/8\left[\frac{4}{3}\pi n^3\right] \longrightarrow (2)$$

- radius 'n+dn' corresponding to energy E+dE ;

$$= \frac{1}{8}\left[\frac{4}{3}\pi(n+dn)^3\right] \longrightarrow (3)$$

Subtracting eqn (3) and eqn (2)

$$N(E)dE = 1/8\left[\frac{4}{3}\pi(n+dn)^3\right] - 1/8\left[\frac{4}{3}\pi n^3\right]$$

$$= 1/8\left(\frac{4\pi}{3}\right)\left[(n+dn)^3 - n^3\right]$$

$$N(E)dE = \left(\frac{\pi}{6}\right)\left[n^3 + dn^3 + 3n^2dn + 3ndn^2 - n^3\right]$$

$$\left[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2\right]$$

Since dn is very small, the higher Powers dn^2 and dn^3 are neglected.

$$N(E)\,dE = \frac{\pi}{6}3n^2dn$$

$$N(E)\,dE = \frac{\pi}{2}n^2dn$$

$$N(E)dE = \frac{\pi}{2}n(ndn) \longrightarrow (4)$$

$$E = \frac{n^2h^2}{8ma^2} \longrightarrow (5) \qquad \left(n^2 = n_x^2 + n_y^2 + n_z^2\right)$$

$$n^2 = \frac{8ma^2E}{h^2} \longrightarrow (6)$$

Taking square root of eqn (6)

$$n = \left(\frac{8ma^2E}{h^2}\right)^{1/2} \longrightarrow (7)$$

Differentiating the eqn (6)

$$2ndn = \frac{8ma^2dE}{h^2}$$

$$ndn = \frac{8ma^2dE}{2h^2} \longrightarrow (8)$$

Substituting eqns (7) and (8) in eqn (4)

$$N(E)\,dE = \frac{\pi}{2}\left(\frac{8ma^2E}{h^2}\right)^{1/2}\left[\frac{8ma^2dE}{2h^2}\right]$$

$$= \frac{1}{2}\,\frac{\pi}{2}\left(\frac{8ma^2E}{h^2}\right)^{1/2}\left[\frac{8ma^2dE}{h^2}\right]$$

$$= \frac{\pi}{4}\left(\frac{8ma^2E}{h^2}\right)^{1/2}\left(\frac{8ma^2dE}{h^2}\right)$$

$$= \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{1/2}E^{1/2}\left(\frac{8ma^2}{h^2}\right)dE$$

$$N(E)\,dE = \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2}E^{1/2}\,dE \longrightarrow (9)$$

According to pauli's exclusion Principle states that two electrons of opposite spins can occupy each state.

$$N(E)\,dE = 2\times\frac{\pi}{4}\left[\frac{8ma^2}{h^2}\right]^{3/2}E^{1/2}\,dE$$

$$= \frac{\pi}{2}\,\frac{(8m)^{3/2}}{(h^2)^{3/2}}\,(a^2)^{3/2}\,E^{1/2}\,dE$$

$$= \frac{\pi}{2}\,(8m)^{3/2}\left(\frac{a^3}{h^3}\right)E^{1/2}\,dE$$

$$N(E)\,dE = \frac{\pi}{2}\left(\frac{a^3}{h^3}\right)(8m)^{3/2}E^{1/2}\,dE \longrightarrow (10)$$

$$(8m)^{3/2} = (8m)^1(8m)^{1/2}$$

$$= 4\times 2m^1\,(4\times 2m)^{1/2}$$

$$= 4\times 2m^1\,(2^2\times 2m)^{1/2}$$

$$= 4\times 2\times(2m)^{3/2}$$

$$= 8\,(2m)^{3/2}$$

$$N(E) = \frac{\pi}{2}\left(\frac{a^3}{h^3}\right)8\,(2m)^{3/2}E^{1/2}\,dE$$

$$= \frac{\pi a^3}{h^3}\,4\,(2m)^{3/2}E^{1/2}\,dE$$

$$N(E)\,dE = \frac{4\pi}{h^3}\,a^3(2m)^{3/2}E^{1/2}\,dE \longrightarrow (11)$$

Density of states is given by the number of energy states per unit volume

$$Z(E)\,dE = \frac{N(E)\,dE}{V} \longrightarrow (12)$$

$$Z(E)\,dE = \frac{\frac{4\pi}{h^3}\,a^3(2m)^{3/2}E^{1/2}\,dE}{a^3}$$

$$[V = a^3]$$

$$\boxed{Z(E)\,dE = \frac{4\pi}{h^3}\,(2m)^{3/2}E^{1/2}\,dE}$$

⑤ Write the Postulates, merits and demerits of quantum theory and derive the expression for electron in metals three dimensional potential well.

i) Quantum free Electron Theory

- This theory uses quantum concepts and hence it is known as quantum free electron theory.

Postulates of Quantum free Electron theory

- Potential energy of an electron is uniform.
- The electrons have wave nature.
- The allowed energy levels of an electron are quantized.
- Electrons move freely within the metal and they are not allowed to leave the metal due to potential barrier at its surface.
- Free electrons obey Fermi Dirac Statistics.

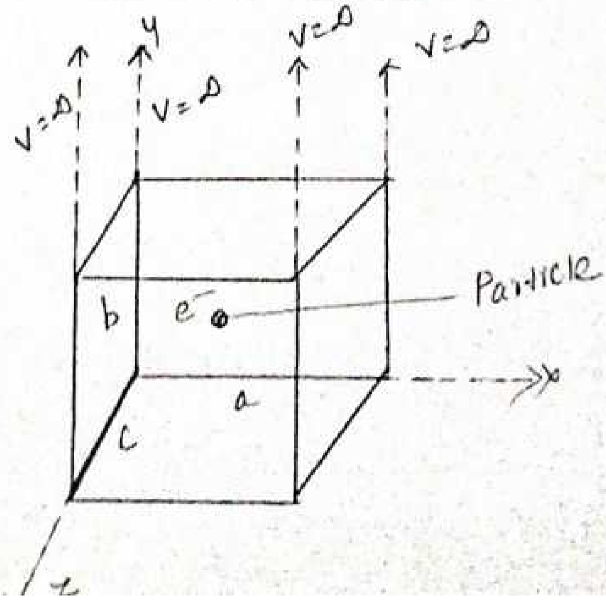
Merits of Quantum free electron theory

- Theory treats the electron quantum mechanically rather than classically.
- It explains the electrical conductivity, thermal conductivity, specific heat capacity of metals, photoelectric effect and compton effect etc.

Demerits of Quantum free Electron theory

- It explains most of the physical properties of the metals.
- It fails to state the difference between Conductor, Semiconductor and insulator.
- It fails to explain the positive value of Hall Coefficient.

ii) Infinite potential well - 3D



- use three quantum numbers n_x, n_y, n_z, to the three coordinate axes namely x, y, z.
- a, b, c - lengths along x, y, z axes.

Energy of the particle = $E_x + E_y + E_z$

$$E_{n_x n_y n_z} = \frac{n_x^2 h^2}{8ma^2} + \frac{n_y^2 h^2}{8mb^2} + \frac{n_z^2 h^2}{8mc^2}$$

If $a = b = c$, in cubical box

$$E_{n_x n_y n_z} = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{a^2} + \frac{n_z^2}{a^2}\right]$$

$$E_{n_x n_y n_z} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2 + n_z^2\right]$$

The wave function of the particle

$$\psi_{n_x n_y n_z} = \sqrt{2/a}\,\sin\left(\frac{n_x \pi x}{a}\right) . \sqrt{2/b}\,\sin\frac{n_y \pi y}{b} . \sqrt{2/c}\,\sin\left(\frac{n_z \pi z}{c}\right)$$

$$\psi_{n_x n_y n_z} = \sqrt{\frac{2}{a} \times \frac{2}{b} \times \frac{2}{c}}\,\sin\frac{n_x \pi x}{a}\,\sin\frac{n_y \pi y}{b}\,\sin\frac{n_z \pi z}{c}$$

$$\boxed{\psi_{n_x n_y n_z} = \sqrt{\frac{8}{abc}}\,\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right)\sin\left(\frac{n_z \pi z}{c}\right)}$$

Example

$n_x = 1,\ n_y = 1,\ n_z = 2$

$n_x^2 + n_y^2 + n_z^2 = 1^2 + 1^2 + 2^2$

$= 1 + 1 + 4$

$n = 6$

$n_x = 1,\ n_y = 2,\ n_z = 1$

$n_x^2 + n_y^2 + n_z^2 = 1^2 + 2^2 + 1^2$

$= 1 + 4 + 1$

$n = 6$

$n_x = 2,\ n_y = 1,\ n_z = 1$

$n_x^2 + n_y^2 + n_z^2 = 2^2 + 1^2 + 1^2$

$= 4 + 1 + 1$

$n = 6$

$$E_{112} = E_{121} = E_{211} = \frac{6h^2}{8ma^2}$$

The corresponding wave funct ion is

$$\psi_{112} = \sqrt{\frac{8}{a^3}}\,\sin\frac{\pi x}{a}\,\sin\frac{\pi y}{b}\,\sin\frac{2\pi z}{c}$$

$$\psi_{121} = \sqrt{\frac{8}{a^3}}\,\sin\frac{\pi x}{a}\,\sin\frac{2\pi y}{b}\,\sin\frac{\pi z}{c}$$

$$\psi_{211} = \sqrt{\frac{8}{a^3}}\,\sin\frac{2\pi x}{a}\,\sin\frac{\pi y}{b}\,\sin\frac{\pi z}{c}$$

⑥ Derive an expression for the effective mass of an electron moving in energy bands of a Solid.

The mass acquired by an electron when it is accelerated in a periodic potential is called effective mass of an electron. It is denoted by m^*.

According to wave mechanics a particle moving with a velocity ν is equivalent to a wave packet moving with a group velocity ν_g

$$\nu_g = \frac{d\omega}{dk} \longrightarrow (1)$$

ω - angular frequency of the electron

k - wave Vector

$$E = h\nu$$

$$E = \frac{h\omega}{2\pi}$$

$$E = \hbar\omega$$

$$\left(\omega = 2\pi\nu \quad \nu = \frac{\omega}{2\pi} \quad \hbar = h/2\pi \right)$$

$$\omega = \frac{E}{\hbar} \longrightarrow (2)$$

Substituting (2) in (1)

$$\nu_g = \frac{d}{dk}\left(\frac{E}{\hbar}\right)$$

$$\nu_g = \frac{1}{\hbar}\frac{dE}{dk}$$

$$a = \frac{d(\nu_g)}{dt}$$

$$= \frac{d}{dt}\left(\frac{1}{\hbar}\left(\frac{dE}{dk}\right)\right]$$

$$= \frac{1}{\hbar}\frac{d^2E}{dk^2}\cdot\frac{dk}{dt}$$

Momentum (P) of an electron inside the crystal

$$P = h/\lambda$$

$$= \frac{h}{2\pi}\frac{2\pi}{\lambda} \qquad \left(k = \frac{2\pi}{\lambda}\right)$$

$$= \hbar k \longrightarrow (5)$$

Differentiating the eqn (5) with respect to t

$$\frac{dP}{dt} = \hbar\frac{dk}{dt} \qquad \left[F = dP/dt\right)$$

$$F = \hbar\frac{dk}{dt}$$

$$\frac{dk}{dt} = \frac{F}{\hbar} \longrightarrow (6)$$

Substituting eqn (6) in eqn (4)

$$a = \frac{1}{\hbar}\frac{d^2E}{dk^2}\cdot\frac{F}{\hbar}$$

$$= \frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\cdot F$$

$$F = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}\right] a \longrightarrow (7)$$

$$a = \frac{eE}{m^*} = \frac{F}{m^*}$$

$$F = m^* a \quad \longrightarrow (8) \quad \boxed{F = eE}$$

Comparing the eqns (7) and (8)

$$m^* a = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \right] a$$

$$m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$$

Special Cases

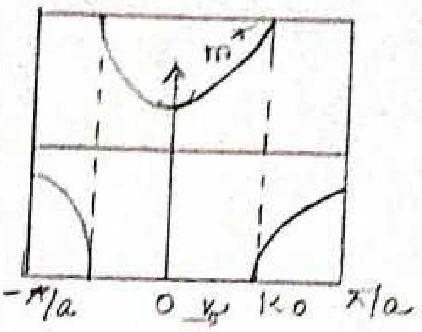
<u>Case (i)</u> If $\frac{d^2E}{dk^2}$ is positive, then m^* is also positive

<u>Case (ii)</u> If $\frac{d^2E}{dk^2}$ is negative, then m^* is also negative

<u>Case (iii)</u> If $\frac{d^2E}{dk^2}$ is more, then the electrons behave as light particle

<u>Case (iv)</u> If $\frac{d^2E}{dk^2}$ is very small, then the electrons behave as heavy particle.

Variation of m^* with k.

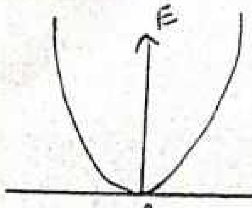


- The variation of m^* with k.
- Value of k increases, m^* increases, maximum value at the point of inflection on the E-k curve
- Above the point of inflection m^* is negative, k tends to π/a, decreases to a small negative value.

⑦ Explain the origin of Energy bands in solids and explain its classification

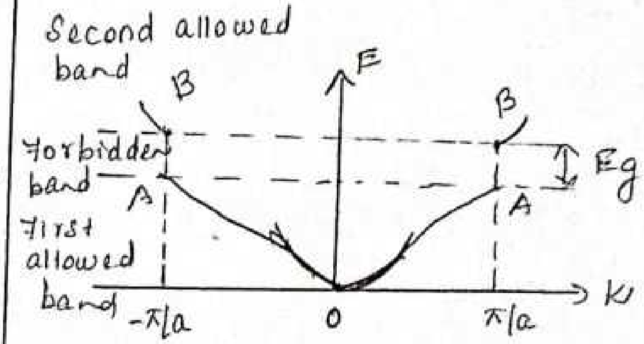
Bloch theorem

- A Solution this problem is given by the zone theory or band theory of Solids.
- The effect of the periodic lattice field on the motion of the electrons leads to the zone or band theory of Solids
- According to zone theory, the electrons move in a periodic field provided by the lattice.
- The potential of the Solid varies periodically with the periodicity of the space lattice.
- The potential energy of the electron is zero near the nucleus of the +ve ion in the lattice.
- when maximum half way between the adjacent nuclei which are seperated by the interatomic spacing distance 'a'
- This model was first postulated by kronig and Penny.



classical free Electron theory (1K)

- In classical theory, we can get a Parabola, when we plot the curve between the electron's energy and its momentum.
- since the curve is parabola the energy varies continuously.



Kronig - Penney model energy curve.

- By Kronig penney model we can get a parabola with some discontinuities.

Consider the Schroedinger eqn for one dimensional periodic Potential field (v(x)) proposed by Kronig and Penny.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(E - V(x)\right)\psi = 0.$$

The solutions of this eqn

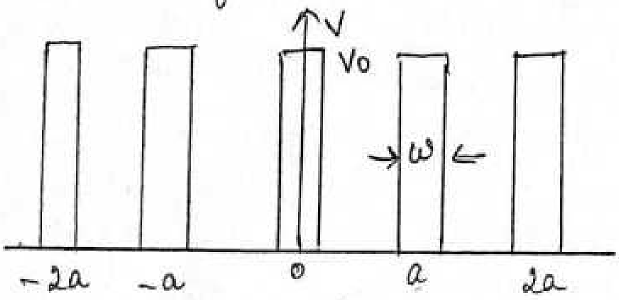
were shown by Bloch to have the form

$$\psi(x) = U_k(x)\, e^{ikx}$$

$U_k(x)$ is periodic with the periodicity of lattice

$$U_k(x+a) = U_k(x)$$

The form of $U_k(x)$ depends on the exact nature of the potential field.



- V_0 increases, the width of the barrier decreases so that the product V_0 remains constant.

$$\cos ka = \frac{d\psi}{dx} \quad P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

$$P = \frac{4\pi^2 m a}{h^2} V_0 w$$

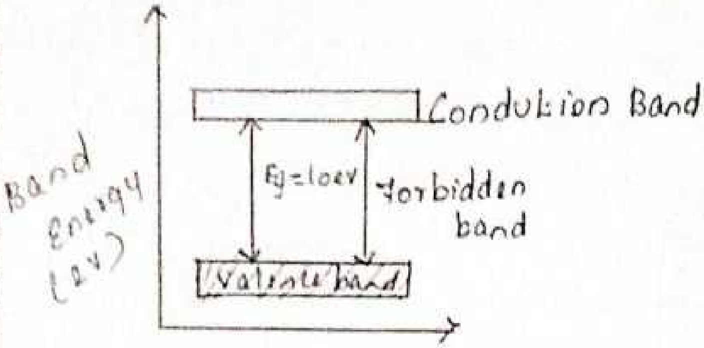
$$\alpha = \frac{2\pi}{h}\sqrt{2mE}$$

Formation of energy Bands in Solids

- A set of such closely spaced energy levels is called an energy band.
- A solid contains an enormous number of atoms packed closely together.
- An electron in a solid can have only those discrete energies lie within the energy bands, called allowed energy bands
- Lower completely filled band is valence band and upper unfilled band is called conduction band.
- The band formed by a series of energy level containing the valence electrons known as valence band.
- The energy gap between the valence band and conduction band is called the forbidden energy gap or forbidden band.

classification of Metals, Semiconductors and Insulators

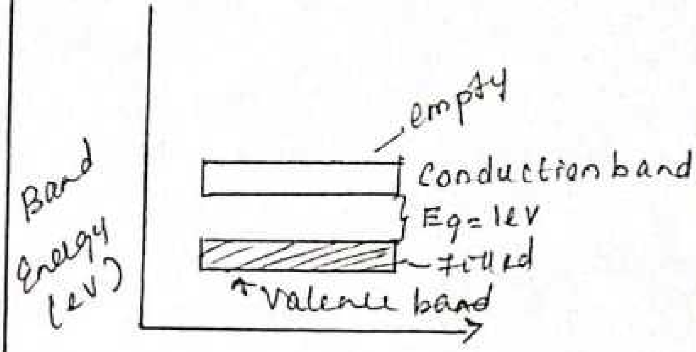
Insulators



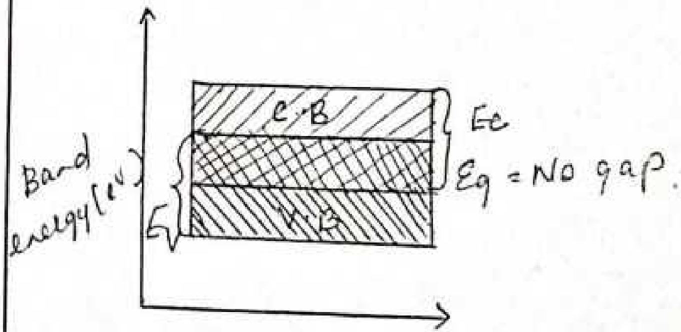
- The energy gap between conduction band and valence band is very high and above 1 eV.
- forbidden energy band is very wide.
- Electrons cannot jump from valence band to conduction band.
- The conduction band is completely vacant and valence band is completely filled.

Semiconductors

- forbidden gap is very small.
- *Examples* : Germanium, Silicon
- Energy gap between conduction band and valence band is very small, and about 0.5 eV to 1eV
- As temperature increases, the bonds in the valence band break up and the created electrons move from valence band to the conduction band.
- Conduction band is partially filled and valence band is partially vacant.



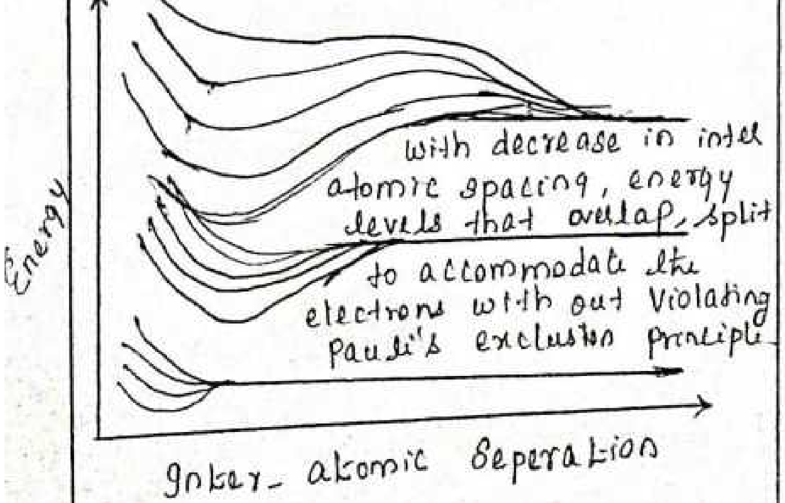
Insulators



- No forbidden gap, both valence and conduction bands overlap each other.
- As temperature increases, the electrical conduction decreases because mobility decreases due to large number of collisions with ions.

⑧ Write notes in Tight Binding approximation

- In Solid, ionic cores at fixed lattice locations and free electron gas enveloping these ionic cores.
- Ionic cores are 'tightly bound' to their lattice locations.
- The electrons are 'free' to run through the extent of the solid. This is called the '<u>Free electron approximation</u>'
- Atoms are free to begin with while the electrons are tightly bound to the atom.
- 'Tight binding approximation' - highlighting the status of the electrons at the start of the model.



- when the atoms are far apart, all the bound electrons associated with each atom have fixed energy levels.
- Solid starts using atoms of the same element.
- Atom will be identical,
- when the atoms get close enough the outer shell electrons begin to overlap with each other.
- The splitting of energy levels occurs because electrons obey the pauli's exclusion principle.
- At each energy, the level will split to enough new energy levels so as to accomodate the electrons of all the atoms in the solid taken together.

Unit - II

Semiconductor Physics

Intrinsic Semiconductors - Energy band diagram - direct and indirect band gap Semiconductors - Carrier Concentration in intrinsic Semiconductors - extrinsic Semiconductors - Carrier Concentration in N type and P type Semiconductors - Variation of carrier Concentration with temperature - Variation of Fermi level with temperature and impurity Concentration - carrier transport in Semiconductor : random motion, drift mobility and diffusion - Hall effect and devices - ohmic Contacts - Schottky diode.

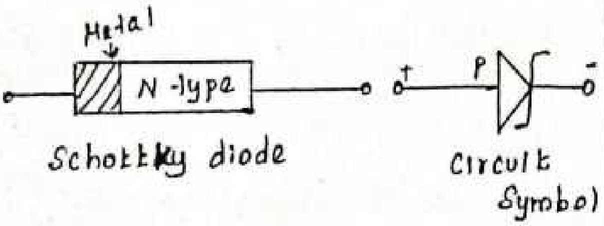
---

① Define construction and working of a Schottky diode.

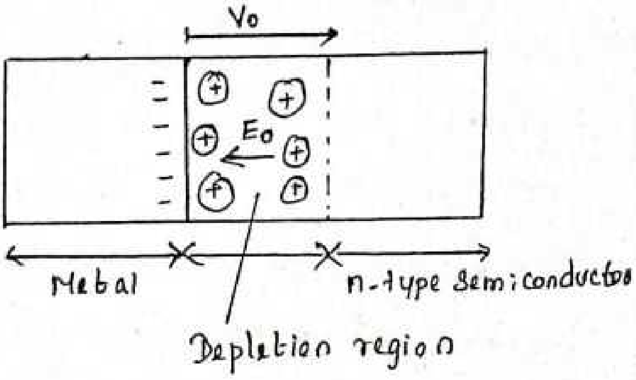
Definition :

It is a junction formed between a metal and n-type Semiconductor.

When the metal has a higher work function than that of n-type Semiconductor then the junction formed is called schottky diode.

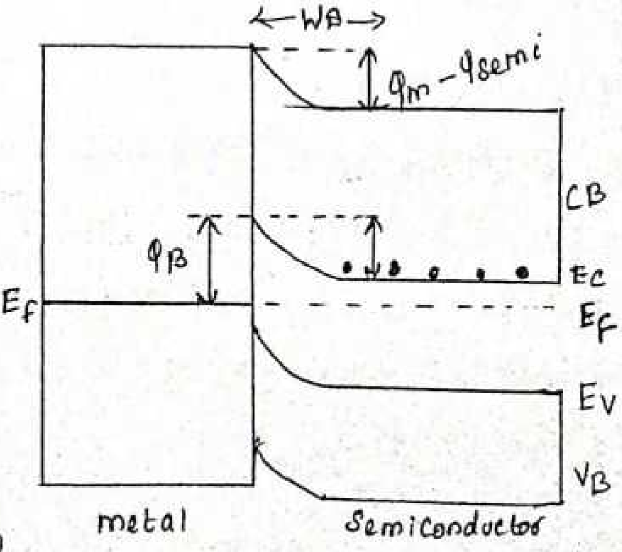


The electrons in the conduction level of the semiconductor move to the empty energy states above the Fermi level of the metal.



- Positive charge on the Semiconductor and a negative charge on the metal. This leads to a Contact potential.

Energy band diagram



- when a Schottky junction is formed between metal and Semiconductor, Fermi lines up.
- positive potential is formed on the semiconductor side.
- Formation of a depletion region of width W_D.
- Bands bend up in the direction of the electric field produced in depletion region.

$$\boxed{eV_0 = \varphi_m - \varphi_{semi}}$$

The Conback potential formed further motion of the electrons between the metal and Semiconductor. This is called the Schottky barrier φ_B

Working.

Voltage is applying in two ways

a) Forward bias

b) Reverse bias

a) Forward bias

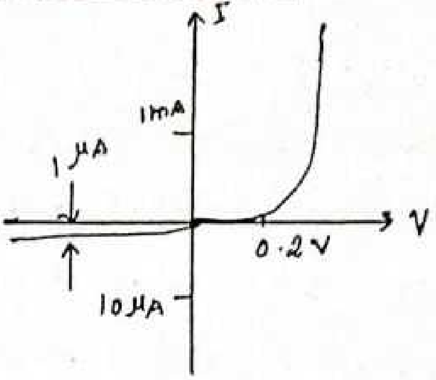
In this bias, metal is Connected to positive terminal and n-type Semiconductor is Connected to negative terminal of the battery.

b) Reverse bias

In reverse bias, metal is Connected to negative terminal and n-type Semiconductor to positive terminal of the battery.

- Increases the width of depletion region
- Schottky junction acts as a rectifier.

V-I Characteristics



- Exponential increase in Current in the forward bias while there is a very Small Current in reverse bias.

Advantages of Schottky diode

- Stored charges or depletion region is negligible.
- very low capacitance.
- depleting region is negligible (fast recovery time)
- high efficiency
- high frequencies
- Produces less noise

Applications of Schottky diode

- used for rectification of signals.
- used in switching device at frequencies of 20 GHz
- used in radio frequency applications
- used in power supplies
- used to detect signals.
- used in logic circuits.

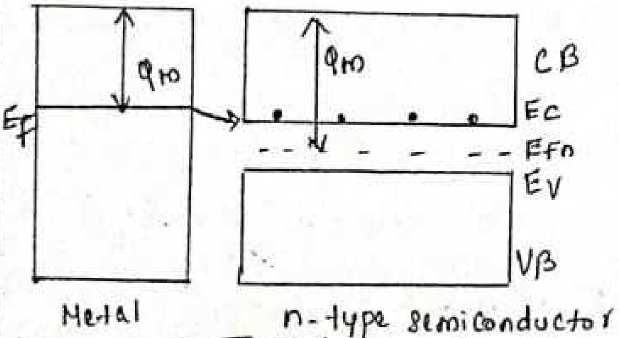
---

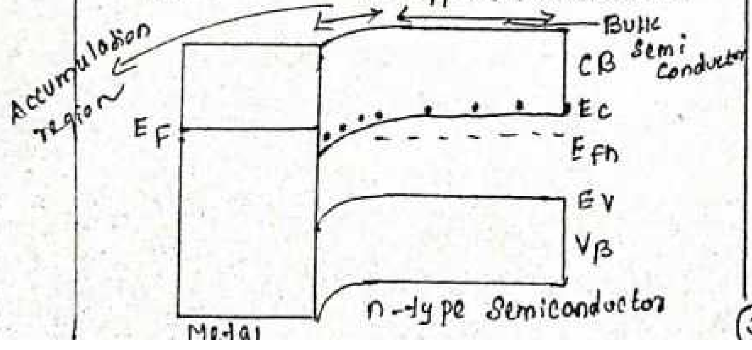
② Write a note on ohmic contact

Definition

An ohmic contact is a type of metal semiconductor junction. It is formed by a contact of a metal with a heavily doped semiconductor.

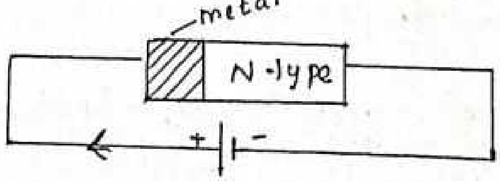
When the semiconductor has a higher work function than that of metal, then the junction formed is called the ohmic junction





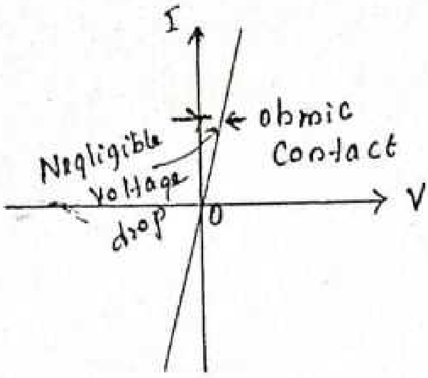
Working

- At equilibrium, the electrons move from the metal to the empty states in the conduction band of semiconductor.
- There is an accumulation region near the interface.
- The accumulation region has a higher conductivity than the bulk semiconductor due to his higher concentration of electrons.
-



- The resistivity is determined by the bulk resistivity of the semiconductor.

V-I characteristics



The current is directly proportional to the potential across the junction

- ohmic contacts are non-rectifying

Applications

The use of ohmic contacts is to Connect one semiconductor device to another, an IC or to connect an IC to its external terminals.

---

③ Derive an expression for electrical conductivity of intrinsic Semiconductor

If the density of free electrons in the material is n, the net charge available per unit volume of the material for the Conduction is equal to ne, where e is the charge of the electron.

when an external field E is applied, the electrons move with a drift Velocity V_{dn}

$$V_{dn} = \mu_n E \longrightarrow (1)$$

μ_n - mobility of electron

The drift Current density J_n due to electrons is defined as the charge flowing across unit area of cross-Section per unit time due to their drift an electrical field E.

$$J_n = n e V_{dn} \longrightarrow (2)$$

σ_n - conductivity of a semiconductor of a free electrons

J_n - Current density

E is applied electric field

$$J_n = \sigma_n E \longrightarrow (3)$$

$$\sigma_n = \frac{J_n}{E} = \frac{n e V_{dn}}{E} \longrightarrow (4)$$

Substituting eqn (1) in eqn (4)

$$\sigma_n = \frac{n e \mu_n E}{E}$$

$$\sigma_n = n e \mu_n \longrightarrow (5)$$

P - number of holes

σ_p - conductivity due to the drift holes

$$\sigma_p = P e \mu_p \longrightarrow (6)$$

μ_p - mobility of holes.

Total conductivity σ due to a free electrons and holes

$$\sigma = \sigma_n + \sigma_p$$

$$\sigma = ne\mu_n + pe\mu_p$$

$$\sigma = e\left(n\mu_n + P(\mu_p)\right) \longrightarrow (1)$$

σ - total conductivity of the material.

For the intrinsic semiconductor which contains the same number of free electrons and holes $n = p = n_i$

From eqn (1)

$$\sigma_i = e(n_i\mu_n + n_i\mu_p)$$

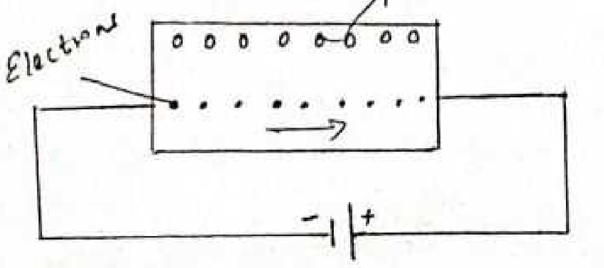
$$\boxed{\sigma_i = en_i(\mu_n + \mu_p)}$$

④ Write down the expression for drift current and diffusion currents

Drift Current

Definition

The electric current produced due to the motion of charge carriers under the influence of an external electric field is known as drift current.



The charge carriers are forced to move in a particular direction due to the electric field. This is known as the drift motion and the current is known as drift current.

Drift current density in a semiconductor due to electrons

$$J_n(\text{drift}) = n\mu_n eE \longrightarrow (1)$$

Drift current density due to hole

$$J_p(\text{drift}) = p\mu_p eE \longrightarrow (2)$$

where

n and p - number of electrons and holes per unit volume.

μ_n, μ_p - mobilities of electrons and holes

e - charge of electrons

E - electric field.

So total drift current density

$$J = J_n(\text{drift}) + J_p(\text{drift})$$

$$\boxed{J = ne\mu_n E + pe\mu_p E} \longrightarrow (3)$$

For intrinsic semiconductor

$$\boxed{J = n_i e(\mu_n + \mu_p)E}$$

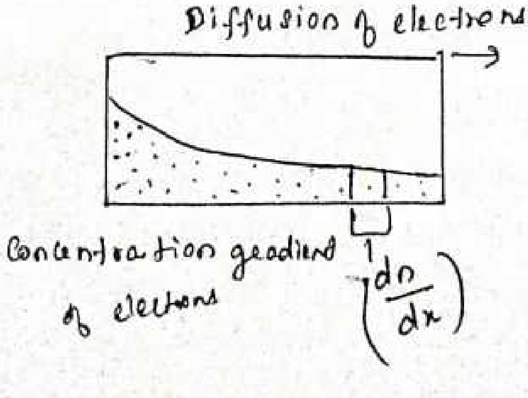
$$(\because n = p = n_i)$$

Diffusion current

Definition

The non-uniform distribution of charge carriers creates the regions of uneven concentrations in the semiconductor.

The charge carriers move from the regions of higher concentration to the regions of lower concentration. This process is known as diffusion. The current is known as diffusion current.



Consider a semiconductor having a concentration gradient of electrons $\frac{dn}{dx}$ within the semiconductor.

- Electrons diffuse from high concentration to low concentration due to the concentration gradient.

Rate of flow of electrons through unit area $\propto -\left(\frac{dn}{dx}\right)$

negative sign denotes the electrons are diffusing from higher concentration to lower concentration region

Rate of flow of electrons through unit area $= -D_n\left(\frac{dn}{dx}\right)$

where

D_n - Proportionality constant known as diffusion coefficient of electrons.

Rate of flow of electrons through unit area

$$= -e \times -D_n\left(\frac{dn}{dx}\right)$$

Rate of flow of electrons

through unit area is the diffusion current density of electrons J_n(diffusion)

$$J_n(\text{diffusion}) = e D_n \left(\frac{dn}{dx}\right)$$

The diffusion current density of holes is given by

$$J_p(\text{diffusion}) = -e D_p \left(\frac{dp}{dx}\right)$$

where D_p is diffusion constant of holes.

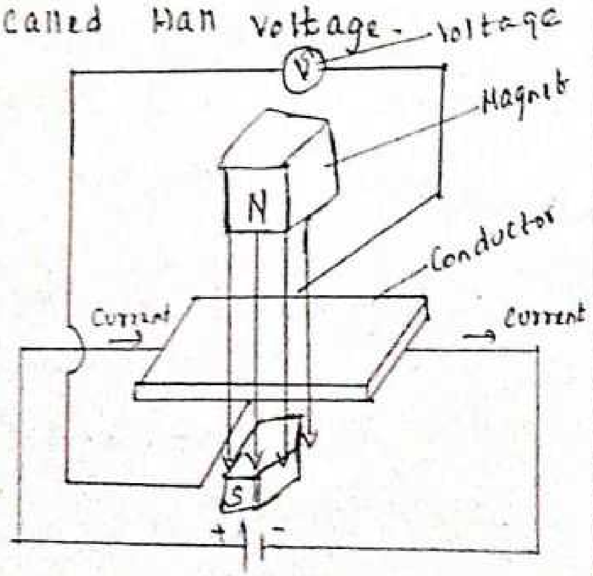
---

⑧ Discuss about Hall Effect and Hall devices

Statement

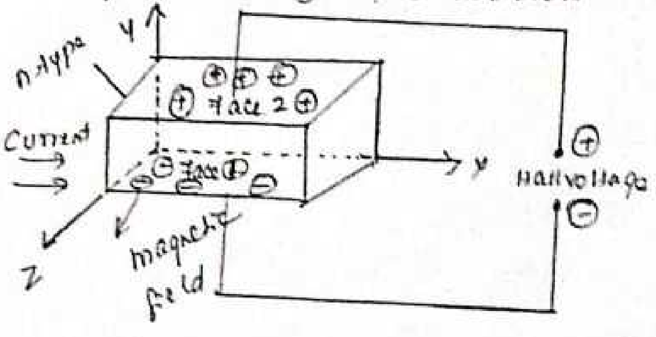
when a Conductor Carrying a current (I) is placed perpendicular to a magnetic field (B), a Potential difference is produced inside the conductor in a direction perpendicular to both current and magnetic field.

This phenomenon is known as Hall effect. The voltage thus generated is called Hall voltage.



Hall effect in n-type Semiconductor

- Consider a n-type Semiconductor in the form of a rectangular slab.
- Current flows in x-direction
- magnetic field B is applied in z-direction.
- Due to Hall effect, voltage is developed along Y-direction



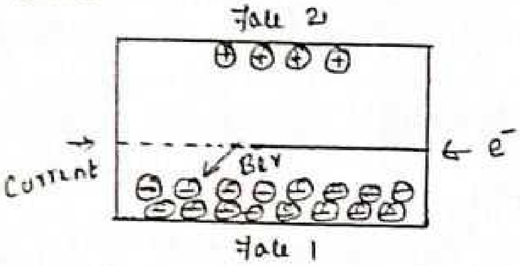
The current flow is entirely due to the flow of electrons moving from right to left along x-direction

⑦

when a magnetic field (B) is applied in z-direction, then the electrons moving with velocity v experience a downward force

Downward force experienced by the electrons $= Bev \longrightarrow (1)$

- Deflects the electrons in downward direction.
- Accumulation of negative charge on the bottom face of the slab.



- a potential difference is developed between top and bottom faces of the slab.
- Potential difference produces an electric field E_H in negative Y-direction. It is called Hall field.
- electric field develops a force (Lorentz force)

upward force acting on each electron $= eE_H \longrightarrow (2)$

At equilibrium, downward force balances upward force

$$Bev = eE_H$$

$$E_H = Bv \longrightarrow (3)$$

The current density (J_x) along x-direction is related to velocity v as

$$J_x = -nev \longrightarrow (4)$$

n - concentration electrons.

$$v = \frac{-J_x}{ne} \longrightarrow (5)$$

Substituting eqn (5) in eqn (3)

$$E_H = \frac{-BJ_x}{ne} \longrightarrow (6)$$

$$E_H = R_H J_x B \longrightarrow (7)$$

$$R_H = -\frac{1}{ne} \quad \text{(for electrons)}$$

$$\boxed{R_H = \frac{E_H}{J_x B}} \longrightarrow (8)$$

R_H - constant and it is known as Hall coefficient.

The negative sign indicates the electric field is developed in negative Y-direction.

<u>Hall effect in p-type Semi Conductor</u>

For p type semiconductor

$$E_H = R_H J_x B$$

where Hall coefficient

The positive sign indicates the electrical field is developed in positive Y-direction.

Hall Coefficient interms of Hall Voltage

If t is the thickness of the sample and V_H is the voltage.

$$V_H = E_H t \longrightarrow (1)$$

where

E_H - Hall field.

Substituting eqn (1) in eqn (8)

$$V_H = R_H J_x B t \longrightarrow (2)$$

If b is breadth of the sample

Cross sectional area of the sample (A) = Breadth (b) x Thickness (t)

= bt

$$\text{Current density } J_x = \frac{I_x}{\text{Area of the Sample (A)}}$$

$$= \frac{I_x}{bt} \longrightarrow (3)$$

Substituting eqn (3) in eqn (2)

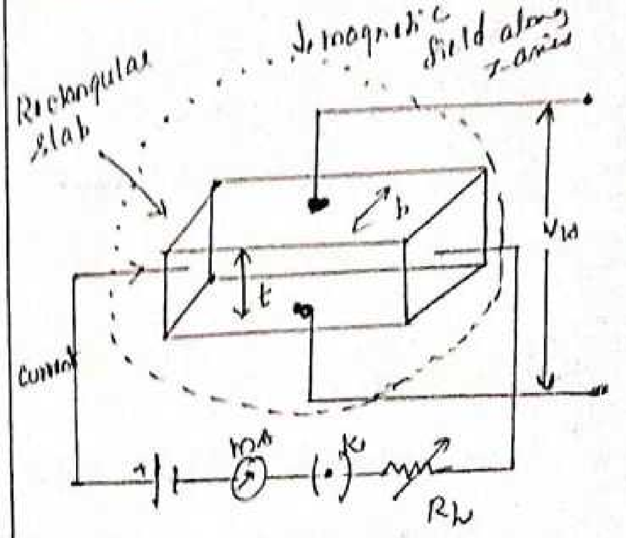
$$V_H = \frac{R_H I_x B t}{bt}$$

$$V_H = \frac{R_H I_x B}{b}$$

Hall coefficient

$$\boxed{R_H = \frac{V_H b}{I_x B}} \longrightarrow (A)$$

Determination of Hall Coefficient



- A semiconductor is taken in the form of a rectangular slab of thickness t and breadth b.
- Current Ix ampere is passed into this sample along x-axis by connecting it to a battery.
- Placed between north and south poles of an electromagnet.
- Magnetic field is applied along Z-axis.
- Due to Hall effect, Hall voltage is developed.

⑨

- Voltage is measured by fixing two probes
- By measuring Hall voltage Hall Coefficient is determined

$$\boxed{R_H = \frac{V_H\, b}{I_x B}}$$

From Hall coefficient, Carrier concentration and mobility can be determined.

Applications of Hall effect

i) Determination of Semi Conductor type

The sign of the Hall coefficient is used to find whether a given semiconductor is n-type or p-type.

ii) Calculation of Carrier Concentration

By measuring Hall Coefficient R_H carrier concentration is determined from the relation

$$\boxed{n = \frac{1}{e R_H}}$$

iii) Determination of mobilit

Electrical Conductivity

$$\sigma_e = n e \mu_e$$

$$\mu_e = \frac{\sigma_e}{ne}$$

$$\boxed{\mu_e = \sigma_e R_H}$$

mobility of charge carriers can be calculated.

Hall Devices

The device which uses the hall effect for its application is known as Hall device

- 3 types

a) Gauss metre

b) Electronic Multiplier

c) Electronic Wattmeter

a) Gauss metre

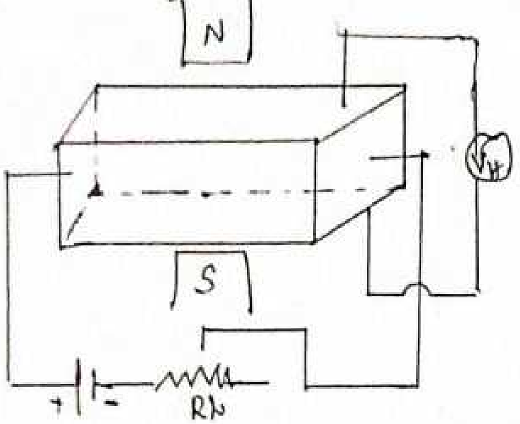
The Hall voltage

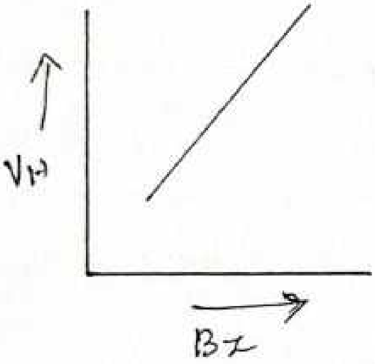
$$V_H = \frac{R_H B_z I_x}{t}$$

$V_H \propto B_z$ for a given hall element

R_H and t are constant.

- Current I through hall element is also kept constant.
- This principle is used in Gauss meter. It is used for measuring magnetic field.





B_z Versus V_H

The voltmeter which is used to measure V_H can be directly caliberated in terms of Gauss.

b) Electronic Multiplier

$$V_H = \frac{R_H B_z I_1}{t}$$

R_H and t - constant

$$V_H \propto B_z I_1$$

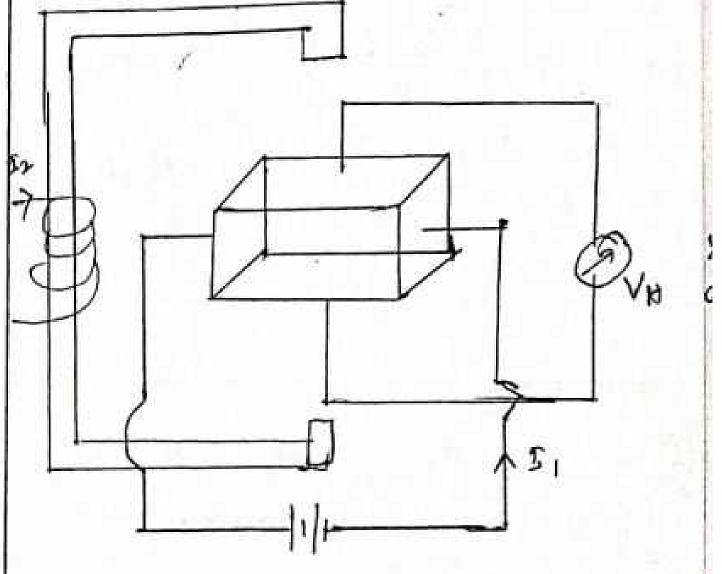
- magnetic field B_z is proportional to current (I_2) through the coil

$$B_z \propto I_2$$

$$V_H \propto I_1 I_2$$

V_H is a measure of the product of two currents.

- Basic Principle used in analog electronic multipliers.

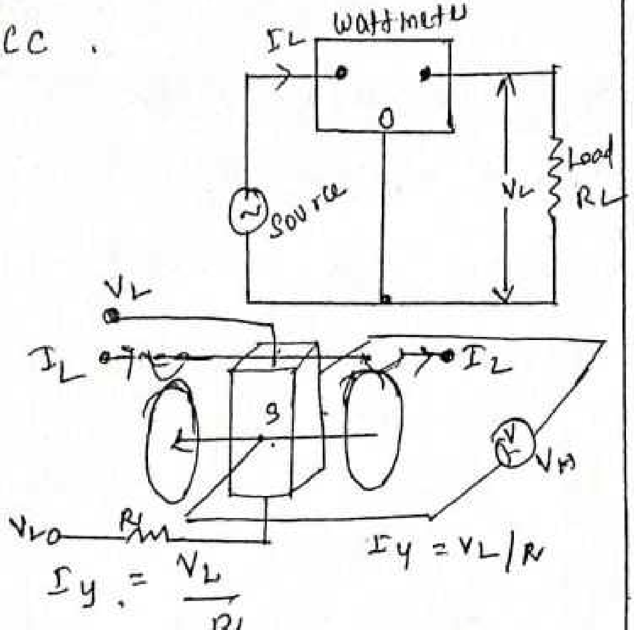


c) Electronic Wattmeter

- Hall effect is used to measure electrical power dissipated in a load.
- The instrument used to measure the power in a circuit using Hall effect Principle is known as Hall effect - Watt meter.

S- Hall effect sample

- magnetic field B_z Produced by the load current I_L Passing through the coils CC.



$$I_y = \frac{V_L}{R}$$

$$I_y << I_L$$

If 't' thickness of the sample the measured Hall voltage

$$V_H = \frac{\underline{R_H B_z I_y}}{t}$$

$$V_H \propto B_z \cdot I_y$$

(R_H and t) constants

$$B_z \propto I_L$$

$$I_y \propto V_L$$

$$\boxed{V_H \propto I_L V_L}$$

- Electric power dissipated by the load.
- Voltmeter measures V_H can be Caliberated to read power directly.

⑥ Derive an expression for Carrier Concentration in intrinsic semiconductors, Density of Electrons in Conduction Band

Definition :

The number of electrons in conduction band per unit Volume of the material is called as electron concentration (n).

The number of electrons per unit volume in Conduction band for energy between E and E+dE

$$dn = Z(E)\,F(E)\,dE \quad \rightarrow (1)$$

$Z(E)\,dE$ - Density of states in energy between E and E+dE

$F(E)$ - Probability of electron occupancy.

Integrating eqn (1) between Energy E_c and $+\infty$

$$\int dn = n = \int_{E_c}^{+\infty} Z(E)\, F(E)\, dE \rightarrow (2)$$

$$Z(E)\,dE = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} E^{1/2}\, dE \rightarrow (3)$$

E is replaced as $(E - E_c)$

$$Z(E)\,dE = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\left(E-E_c\right)^{1/2} dE \rightarrow (4)$$

The probability of electron occupation is given by Fermi distribution function

$$F(E) = \frac{1}{1+e^{(E-E_f)/kT}} \rightarrow (5)$$

Substituting eqns (4) and (5) in (2)

$$n = \int_{E_c}^{+\infty} \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\left(E-E_c\right)^{1/2} \times \frac{1}{1+e^{(E-E_f)/kT}} . dE$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\int_{E_c}^{+\infty} \frac{\left(E-E_c\right)^{1/2}}{1+e^{(E-E_f)/kT}} . dE \rightarrow (6)$$

Since KT is very small and $(E-E_f)$ is greater than kT, $e^{(E-E_f)/kT}$ is very large compared to '1'. Hence '1' from the denominator of eqn (6) is neglected.

$$1+e^{(E-E_f)/kT} \approx e^{(E-E_f)/kT}$$

Eqn (6) becomes

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\int_{E_c}^{+\infty} \frac{\left(E-E_c\right)^{1/2}}{e^{(E-E_f)/kT}} . dE$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\int_{E_c}^{+\infty} \left(E-E_c\right)^{1/2} . e^{-(E-E_f)/kT} . dE$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\int_{E_c}^{+\infty} \left(E-E_c\right)^{1/2} . e^{(E_f-E)/kT} . dE$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} e^{E_f/kT}\int_{E_c}^{+\infty} (E-E_c \quad e^{-E/kT} . dE \rightarrow (7)$$

Substituting above values in eqn (7)

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} e^{E_f/kT}\int_{0}^{\infty} x^{1/2}\, e^{-(E_c+x)/kT} . dx$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} e^{(E_f-E_c)/kT}\int_{0}^{\infty} x^{1/2}\, e^{-x/kT} . dx \rightarrow (8)$$

Using the gamma function

$$\int_0^\infty x^{1/2} e^{-x/kT} dx = \frac{(kT)^{3/2} \pi^{1/2}}{2} \longrightarrow (9)$$

Substituting eqn (9) in eqn (8)

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{E_F - E_c/kT} \cdot \left[\frac{(kT)^{3/2} \pi^{1/2}}{2}\right]$$

$$n = \frac{2\pi}{h^3} (2m_e^*)^{3/2} (kT)^{3/2} \pi^{1/2} \cdot e^{(E_F - E_c)/kT}$$

$$n = \frac{2\pi \pi^{1/2} (2m_e^*)^{3/2} (kT)^{3/2} e^{(E_F - E_c)/kT}}{(h^2)^{3/2}}$$

$$\boxed{n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{E_F - E_c/kT} \longrightarrow (10)}$$

Eqn (10) is the expression for concentration of electrons in the conduction band of intrinsic semiconductor.

---

⑦ **Derive an expression for Density of holes in valence band of intrinsic semiconductor.**

Let dp be the number of holes per unit volume in vacancy band between an energy E and E+dE

$$dp = Z(E)\,(1 - F(E))\,dE \longrightarrow (1)$$

where

Z(E) dE → Density of states in the energy range E and E+dE

$$1 - F(E) = 1 - \frac{1}{1 + e^{(E - E_F)/kT}}$$

$$= \frac{1 - e^{E - E_F/kT} - 1}{1 + e^{E - E_F/kT}}$$

$$1 - F(E) = \frac{e^{E - E_F/kT}}{1 + e^{E - E_F/kT}} \longrightarrow (2)$$

Since E is very small when compared to E_F in valence band, $(E - E_F)$ is a negative quantity, $e^{E - E_F/kT}$ is very small and neglected in the denominator of eqn (2)

(14)

$$1 + e^{E-E_F/kT} \approx 1$$

$$\therefore \; 1 - F(E) = e^{(E-E_F)/kT} \longrightarrow (3)$$

Density of states in valence band

$$Z(E)\,dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} E^{1/2}\,dE \longrightarrow (4)$$

m_h^* is the effective mass of the hole in valence band.

Eqn (4) is replaced as $(E_v - E)$

$$Z(E)\,dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\left(E_v-E\right)^{1/2} dE \longrightarrow (5)$$

Substituting eqns (3) and (5) in (1)

$$dp = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\left(E_v-E\right) e^{(E-E_f)/kT}.\,dE \longrightarrow (6)$$

Integrating eqn (6) between limits $-\infty$ to E_v.

$$\int dp = P = \int_{-\infty}^{E_v} \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} \left(E_v-E\right)^{1/2} e^{(E-E_f)/kT}.\,dE$$

$$P = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} e^{-E_f/kT} \int_{-\infty}^{E_v} \left(E_v-E\right)^{1/2} e^{E/kT}\,dE \rightarrow (7)$$

Substituting the values in eqn (7)

$$dE = -dx$$

$$\left(E_v - E\right) = x$$

$$E = \left(E_v - x\right)$$

$$P = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} e^{-E_f/kT} \int_{\infty}^{0} x^{1/2} e^{(E_v-x)/kT}\,(-dx) \longrightarrow (8)$$

$$P = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} e^{(E_v-E_f)/kT} \int_{0}^{\infty} x^{1/2} e^{-x/kT}\,dx \longrightarrow (9)$$

(-ve sign is omitted by interchanging the limits)

$$\int_{0}^{\infty} x^{1/2} e^{-x/kT}\,dx = \frac{(kT)^{3/2}\pi^{1/2}}{2} \longrightarrow (10)$$

Substituting eqn (10) in eqn (9

$$P = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} e^{(E_v-E_f)/kT} \left[\frac{(kT)^{3/2}\pi^{1/2}}{2}\right]$$

$$p = \frac{2\pi}{h^3}\left(2m_h^*\right)^{3/2}(kT)^{3/2}\pi^{1/2} e^{(E_v-E_f)/kT}$$

$$p = \frac{2\pi\pi^{1/2}\left(2m_h^*\right)^{3/2}(kT)^{3/2} e^{(E_v-E_f)/kT}}{(h^2)^{3/2}}$$

$$\boxed{p = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_v-E_f)/kT}} \longrightarrow (11)$$

Eqn (11) is the expression for the concentration of holes in valence band of intrinsic Semiconductor.

⑧ Intrinsic Carrier concentration

$$n_i = n = p \longrightarrow (1)$$

$$n_i \times n_i = n_i^2 = np \longrightarrow (2)$$

Substituting the expressions of n and p in eqn (2)

$$n_i^2 = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{E_f-E_c/kT} \times 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{E_v-E_f/kT}$$

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^{3}\left(m_e^* m_h^*\right)^{3/2} e^{(E_v-E_c)/kT}$$

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^{3}\left(m_e^* m_h^*\right)^{3/2} e^{-E_g/kT}$$

where

$E_c - E_v = E_g$ forbidden energy gap.

Taking square root on both sides in eqn (3)

$$\left(n_i^2\right)^{1/2} = \left[4\left(\frac{2\pi kT}{h^2}\right)^{3}\left(m_e^* m_h^*\right)^{3/2} e^{-E_g/kT}\right]^{1/2}$$

$$n_i = 4^{1/2}\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(\left(m_e^* m_h^*\right)^{3/2}\right)^{1/2}\left(e^{-E_g/kT}\right)^{1/2}$$

$$n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_e^* m_h^*\right)^{3/4} e^{-E_g/2kT}$$

$$\longrightarrow (A)$$

The eqn (4) is expression for intrinsic carrier Concentration -

(8) Derive an expression for carrier concentration in n-type Semiconductors

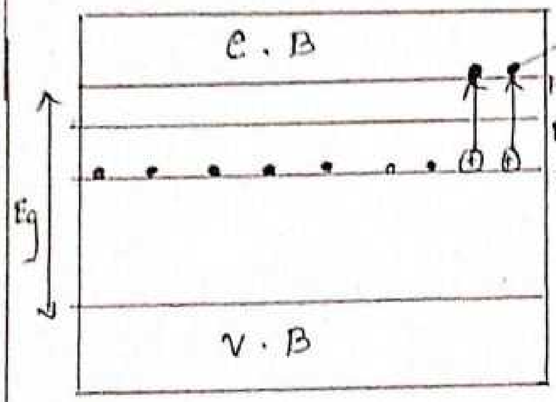
Density of electrons per unit volume in conduction band is given by

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{E_F - E_C/kT} \longrightarrow (1)$$

E_C - Energy corresponding to the bottom most level of Conduction band.

Density of ionised donors

$$= N_d\left[1 - F(E_d)\right]$$



E_d represents the donor energy level

N_d denotes donor Concentration.

$$= N_d\left[1 - \frac{1}{1+e^{(E_d-E_F)/kT}}\right] \longrightarrow (2)$$

$$= N_d\left[\frac{1 + e^{E_d - E_F/kT} - 1}{1+e^{(E_d-E_F)/kT}}\right]$$

$$= N_d\,\frac{(e^{E_d - E_F/kT}}{1+e^{(E_d-E_F)/kT}} \longrightarrow (3)$$

$e^{E_d-E_F/kT}$ is very small in eqn (3) when compared to '1'. Hence it is neglected.

$$1 + e^{(E_d-E_F)/kT} \approx 1$$

Density of ionised donor

$$= N_d\, e^{(E_d-E_F)/kT} \longrightarrow (4)$$

At equilibrium,

Equating (1) & (4)

$$2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} (e^{(E_F-E_C)/kT} = N_d\, e^{(E_d-E_F)/kT} \longrightarrow (5)$$

rearranging the terms

$$\frac{e^{E_F - E_C/kT}}{e^{E_d - E_F/kT}} = \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_f - E_c)/kT} \, e^{-(E_d - E_f)/kT} = \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_f - E_c + E_f - E_d)/kT} = \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}} \longrightarrow (6)$$

Taking log on both sides

$$\log_e e^{(E_F - E_C - E_d + E_f)/kT} = \log_e \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}$$

$$\frac{E_f - E_C - E_d + E_f}{kT} = \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

$$\left[\therefore \log_e e^x = x\right]$$

$$2E_f - (E_C + E_d) = kT \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

or

$$2E_f = E_d + E_C + kT \log_e \left(\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right)$$

$$\boxed{E_F = \frac{E_d + E_c}{2} + \frac{kT}{2} \log_e \left(\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right)}$$

$$\longrightarrow (7)$$

Substituting the expression of E_F from (7) in (1)

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left[\frac{\frac{E_d + E_c}{2} + \frac{kT}{2}\log_e\left\{\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right\} - E_c}{kT}\right]$$

$$\longrightarrow (8)$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left[\frac{E_d + E_c - 2E_c}{2kT} + \frac{1}{2}\log_e\left\{\frac{N_d}{2\left\{\frac{2\pi m_e^* kT}{h^2}\right\}^{3/2}}\right\}\right]$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left[\frac{E_d - E_c}{2kT} + \log_e\left\{\frac{N_d^{1/2}}{\left\{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right\}^{1/2}}\right\}\right]$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{E_d - E_c/2kT} \cdot e^{\log_e \left\{\frac{N_d^{1/2}}{\left\{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right\}^{1/2}}\right\}}$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_d - E_c)/2kT} \cdot \frac{N_d^{1/2}}{2^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4}} \longrightarrow (9)$$

Rearranging the expression (9)

$$n = 2\,\frac{N_d^{1/2}}{2^{1/2}}\,\frac{\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}{\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4}} \cdot e^{(E_d - E_c)/2kT}$$

$$n = 2^{1/2} \times 2^{1/2}\,\frac{N_d^{1/2}}{2^{1/2}} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{-3/4} e^{E_d - E_c/2kT}$$

$$n = 2^{1/2} N_d^{1/2} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} \cdot e^{(E_d - E_c)/2kT} \longrightarrow (10)$$

$$\boxed{n = (2N_d)^{1/2} \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} \cdot e^{-\Delta E/2kT}}$$

where $\longrightarrow (11)$

$\Delta E = E_c - E_d$.

⑩ Derive an expression for concentration of holes in Valence band of P-type Semiconductors

Density of holes per unit volume in Valence band is given by

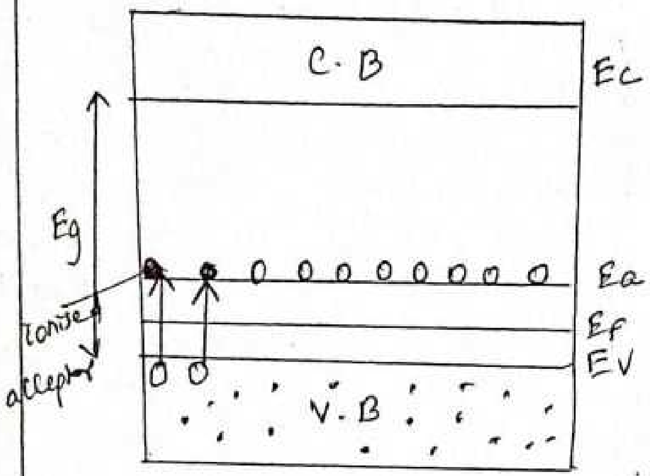
$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_v - E_F)/kT} \longrightarrow (1)$$

$E_v \rightarrow$ Energy corresponding to top most level of valence band.

Density of ionised acceptors

$$= N_a F(E_a) \longrightarrow (1)$$

N_a - the number of acceptor atoms per unit volume.



$$F(E_a) = \frac{1}{1 + e^{(E_a - E_f)/kT}}$$

E_a - acceptor energy level

$F(E_a)$ is probability for finding electron in acceptor energy level.

The eqn (1) becomes density of ionised acceptors

$$= \frac{N_a}{1 + e^{(E_a - E_f)/kT}} \longrightarrow (2)$$

$e^{(E_a - E_f)/kT}$ is a large quantity and thus '1' from the denominator of eqn (2) is neglected.

Eqn (2) is modified as

$$N_a F(E_a) = \frac{N_a}{e^{(E_a - E_f)/kT}}$$

$$N_a F(E_a) = N_a e^{-(E_a - E_f)/kT}$$

Density of ionised acceptors

$$= N_a e^{(E_f - E_a)/kT} \longrightarrow (3)$$

At equilibrium

$$\begin{pmatrix} \text{Density of holes} \\ \text{in valence band} \end{pmatrix} = \begin{pmatrix} \text{Density of} \\ \text{ionised} \\ \text{acceptors} \end{pmatrix}$$

$$2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_v - E_f)/kT} = N_a e^{(E_f - E_a)/kT} \longrightarrow (4)$$

rearranging eqn (4) we have

$$\frac{e^{(E_v - E_f)/kT}}{e^{(E_f - E_a)/kT}} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_v - E_f)/kT} \cdot e^{-(E_f - E_a)/kT} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_v - E_F - E_F + E_a)/kT} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}} \longrightarrow (5)$$

Taking log on both sides in eqn(5)

$$\log_e e^{(E_v - E_F - E_F + E_a)/kT} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}$$

$$\frac{E_v - 2E_F + E_a}{kT} = \log_e\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]$$

$$E_a + E_v - 2E_F = kT\log_e \left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right] \longrightarrow (6)$$

Rearranging

$$2E_F = E_a + E_v - kT\log_e \left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]$$

$$\boxed{E_F = \frac{E_a + E_v}{2} - \frac{kT}{2}\log_e \left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]} \longrightarrow (7)$$

Substituting eqn E_F from (7) in (1)

$$p = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp \left[E_v - \left[\left(\frac{E_a + E_v}{2}\right) - \frac{kT}{2}\log_e \left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]\right]\right]$$

$$p = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp \left[\frac{\left\{E_v - \left(\frac{E_v - E_a}{2}\right) + \frac{kT}{2}\log_e \left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]\right\}}{kT}\right] \longrightarrow (8)$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp \left[\frac{2E_v - E_v - E_a}{2kT} + \frac{1}{2}\log_e \left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]\right]$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp \left[\frac{E_v - E_a}{2kT} + \log_e \left(\frac{\frac{N_a}{2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}}\right)\right]^{1/2}$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp \left[\frac{E_v - E_a}{2kT} + \log_e \left[\frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]\right]$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{E_v - E_a/2kT} . \; e^{\log_e \left[\frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}}\right]}$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{E_v - E_a/2kT} . \; \frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}}$$

$$P = \frac{2\, N_a^{1/2}}{2^{1/2}} \; \frac{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}} . \; e^{(E_v - E_a)/2kT} .$$

$$P = \frac{2^{1/2}\, 2^{1/2}\, N_a^{1/2}}{2^{1/2}} \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} . \left(\frac{2\pi m_h^* kT}{h^2}\right)^{-3/4} e^{E_v - E_a/2kT}$$

$$P = (2N_a)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} e^{E_v - E_a/2kT} \longrightarrow (9)$$

$$\boxed{P = (2N_a)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} e^{-\Delta E/2kT}}$$

where $\Delta E = E_a - E_v$

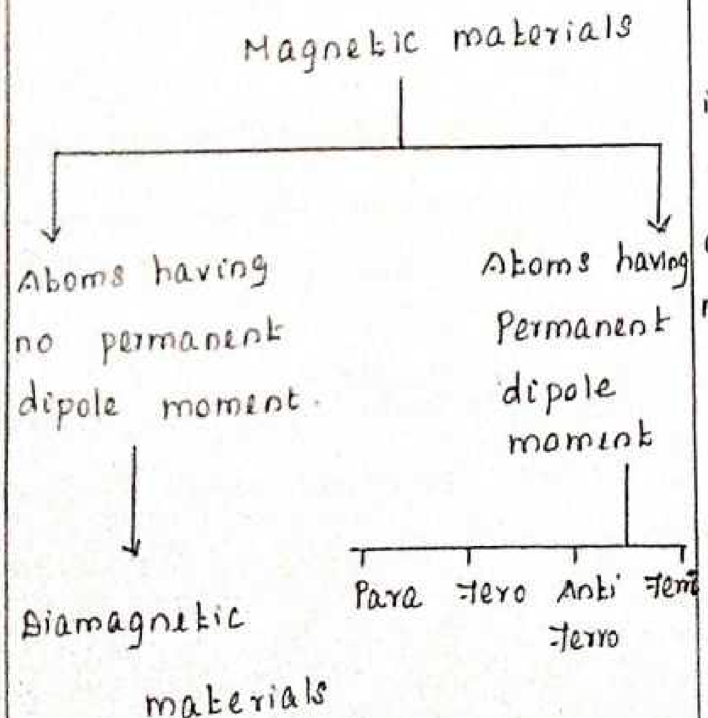
UNIT III

Magnetic Properties of Materials

Magnetic dipole moment - atomic magnetic moments - magnetic permeability and susceptibility - Magnetic material classification : diamagnetism - paramagnetism - ferromagnetism. antiferromagnetism - ferrimagnetism - Ferromagnetism : origin and exchange interaction saturation magnetization and curie temperature - Domain theory - M versus H behaviour - Hard and soft magnetic materials - examples and uses - Magnetic principle in computer data storage - Magnetic hard disc (GMR sensor)

① Discuss about classification of magnetic materials

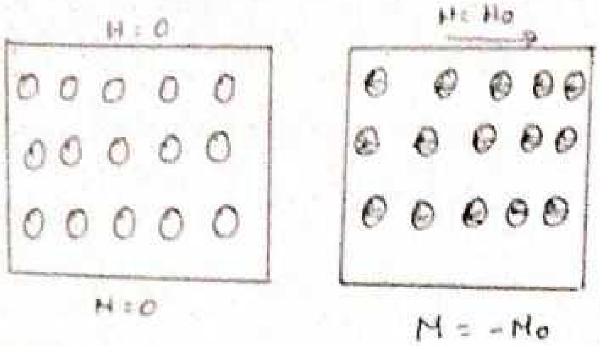
Magnetic materials are classified into two categories based on dipole moment.



i) Diamagnetism

- Diamagnetism is exhibited by all the materials.
- when diamagnetic material is placed in an external magnetic field, the atomic orbits tend to the external magnetic field.
- Atoms acquire an induced magnetic moment.
- As a result, the material becomes magnetised.
- The direction of the induced dipole moment is opposite to that of externally applied magnetic field.

Due to this effect, the material is very weakly repelled in magnetic field. This phenomenon is known as diamagnetism.



Diamagnetic Materials

The materials which exhibit diamagnetism are called diamagnetic materials.

Properties.

i) Repel the magnetic lines of force.

ii) No permanent dipole moment.

iii) magnetic susceptibility is negative.

Example :

Gold, germanium and Silicon.

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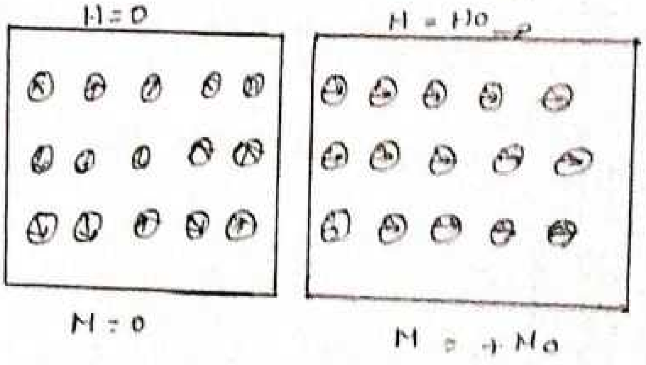
② Paramagnetism

- The magnetic moments are randomly oriented in the absence of an external magnetic field.
- Net magnetic moment is zero.
- Magnetisation of the material is zero.
- When the external magnetic field is applied, the magnetic dipoles tend to align themselves in the direction of the magnetic field.
- Material becomes magnetised.



- This effect is known as paramagnetism.
- The paramagnetic susceptibility varies inversely with temperature.

$$\chi \propto 1/T$$

$$\boxed{\chi = C/T}$$

This is known as Curie's law of paramagnetism.

C is a constant called curie's constant.

Paramagnetic materials.

The magnetic materials which exhibit paramagnetism are called as paramagnetic material.

Properties

i) Attract the magnetic lines of force.

ii) Possess permanent dipole moments.

iii) susceptibility is positive and it depends on temperature

$$\boxed{\chi = \frac{C}{T-\theta}}$$

Example :

Ferric oxide, nickel Sulphate.

③ Ferromagnetism

Certain metals like iron (Fe), cobalt (Co), nickel (Ni) and certain alloys exhibit high degree of magnetisation. These materials show the spontaneous magnetization even in the absence of an external magnetic field. There is a strong internal field with in the material which makes the atomic magnetic moments align with each other. This phenomenon is called ferromagnetism.

Ferromagnetic materials

The materials which exhibit the ferromagnetism are called ferromagnetic materials.

Properties

- All the dipoles are aligned parallel to each other due to the magnetic interaction between the dipoles.
- Have permanent dipole moment. Strongly attracted by the magnetic field.
- Exhibit hysteresis
- Magnetic susceptibility is very high, depends on temperature

$$\boxed{\chi = \frac{C}{T-\theta}}$$ (for $T > \theta$, Para magnetic behaviour

$T < \theta$, Ferromagnetic behaviour.

④ Antiferromagnetism

Antiferromagnetic materials are magnetic materials which exhibit a small positive susceptibility of the order of 10^{-3} to 10^{-5}.

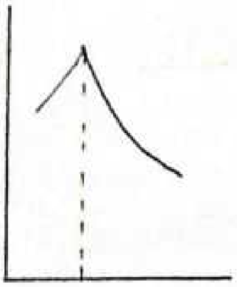
The variation of susceptibility with temperature obeys modified Curie - Weiss law

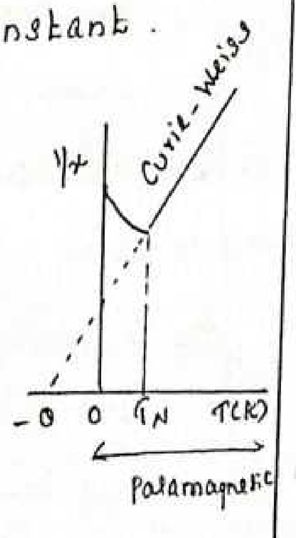
$$\boxed{\chi_{a.f} = \frac{C}{T-(-\theta)} = \frac{C}{T+\theta}}$$

when $T > T_N$

θ - Paramagnetic Curie temperature

C - Curie constant.





Antiferromagnetic materials

Magnetic materials exhibit antiferromagnetism are called as antiferromagnetic materials.

Properties of antiferromagnetic materials.

i) Magnetic dipoles are aligned antiparallel

ii) Depends on temperature

iii) Susceptibility is small and positive

$$\boxed{\chi = \frac{C}{T+\theta}} \quad \text{when } T > T_N$$

T_N - Neel temperature

$$\boxed{\chi \propto T} \quad \text{when } T < T_N$$

⑤ Ferrimagnetism

Crystals possess spontaneous magnetization and exhibit most of the properties of ferromagnetic materials. This uncompensated antiferromagnetism is known as ferrimagnetism.

Ferrimagnetic materials or Ferrites

Materials exhibit ferrimagnetism are called ferrimagnetic materials or ferrites.

Properties of ferrites

- Ferrites has net magnetic moment.
- Above Curie temperature, it becomes para-magnetic and it behaves as ferri magnetic material below Curie temperature.
- Susceptibility of ferrite is very large and positive.
- Depends on temperature.

$$\chi = \frac{C}{T \pm \theta} \quad \text{for } T > T_N$$

- Spin allignment is anti parallel of different magnitudes.
- High permeability
- High resistivity
- Low eddy current loss
- Low hysteresis loss.

Applications of ferrites.

- Hard magnetic ferrites are used in the manufacture of permanent magnets.
- Used in super high frequency technology.
- Used in the production of cores.
- Used in magnetic films.
- Used in information storage devices such as magnetic discs and tapes
- Ferrite rods are used to produce ultrasonics by magnetostriction principle.
- Used in radio receiver to increase sensitivity and selectivity.

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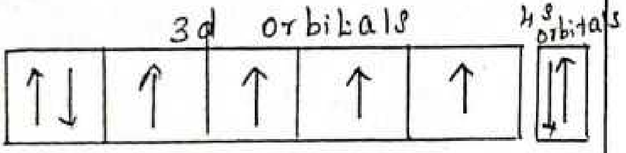
② Write notes on i) origin of ferromagnetism and Exchange Interaction. ii) Saturation magnetization and Curie temperature

i) Origin of ferromagnetism

The ferromagnetic property is exhibited by transition elements like iron, cobalt and nickel and rare earth elements like gadolinium and dysprosium

- possess parallel alignment of dipoles
- magnetic potential energy is very small
- Electronic Configuration of iron is $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^6, 4s^2$
- 3d subshell is an unfilled one. This 3d subshell have five orbitals.



The Pauli's exclusion Principle and electrostatic interaction energy are combined together and continue a new kind of interaction known as exchange interaction. The exchange interaction is a quantum mechanical concept.

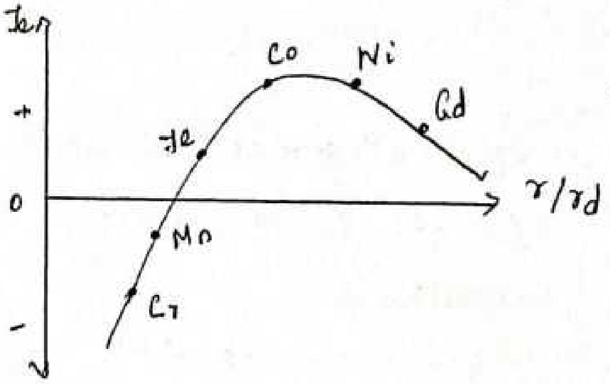
The exchange interaction between any two atoms is given by

$$\boxed{E_{ex} = -J_e S_1 S_2}$$

J_e - numerical value

S_1, S_2 - spin angular momenta

- Exchange integral value is negative for a number of elements.
- antiparallel alignment of dipoles in antiferro magnetic materials.



iron, cobalt nickel and gadolinium - positive

manganese, chromium - Negative

If the ratio, $r/r_d > 3$ the material is ferromagnetic otherwise antiferromagnetic.

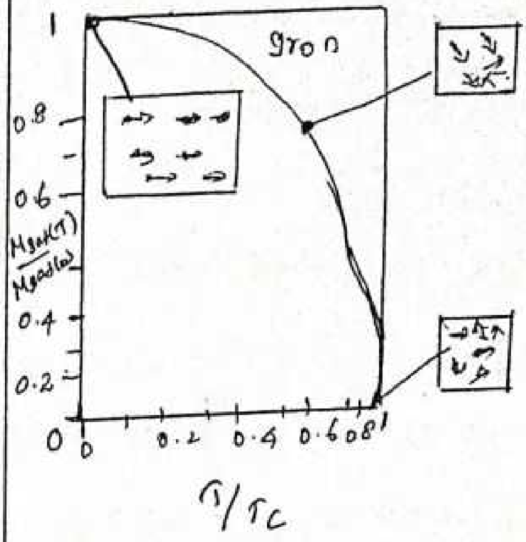
- Manganese is suitable alloyed $r/r_d > 3$ will become ferromagnetic

ii) Saturation magnetization and curie temperature

Definition

The maximum magnetization in a ferromagnet when all the atomic magnetic moments are aligned is called the saturation magnetization M_{sat}.

- Temperature increased, lattice vibrations become more energetic
- Spins cannot align perfectly with each other.
- The ferromagnetic behaviour disappears at a critical temperature called the curie temperature
- Above the curie temperature the ferromagnetic materials behaves like paramagnetic.
- Saturation magnetization M_{sat} decreases from its maximum value $M_{sat}(0)$ at absolute zero temperature to zero at the curie temperature.



- M_{sat} depends on the temperature when M_{sat} is normalized to $M_{sat}(0)$ and temperature is reduced temperature T/T_c at $T/T_c = 1$, $M_{sat} = 0$

Since the curie temperature

$$\boxed{E_{ex} = kT_c}$$

- Magnetic susceptibility of ferromagnetic materials is very large
- Magnetic susceptibility

$$\boxed{\chi = \frac{C}{T - T_c}}$$

C - Curie Constant.

③ Discuss Domain theory of Ferromagnetism, Process and types of Energy

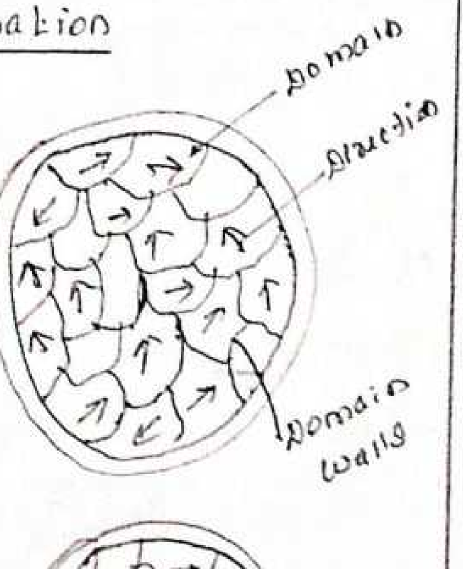
Domain theory

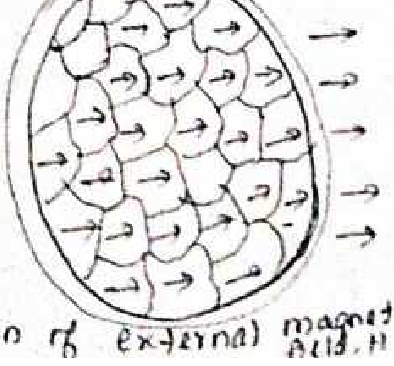
Weiss proposed the concept of domains in order to explain the properties of ferromagnetic materials.

Principle

The group of atomic dipoles organised in tiny bounded regions in the ferromagnetic materials are called magnetic domains

Explanation





Direction of external magnetic field. H ⑧

- Contains a large number of domains
- magnetic moments of the atoms are aligned in same direction.
- domain in a region of the ferromagnetic material in which all the magnetic moments are aligned to produce a net magnetic moment in one direction only.
- own magnetic moment and axis.
- In demagnetized ferromagnetic material, the domains are randomly oriented.
- magnetization of the material as a whole is zero.
- The boundaries seperating the domains are called domain walls.
- Domain walls are thicker than the grain boundaries.

- Domain size can also grow due to the movement of domain walls.
- When a magnetic field is applied externally to a ferromagnetic material, the domains align themselves with field. This results in a large net magnetization of the material.
- The domain walls are also known as Bloch walls.

④ Process of Domain magnetisation

- In an unmagnetised specimen the domains are randomly oriented and the net magnetization is zero.
- There are two possible ways in which the domains are aligned in the external field direction.

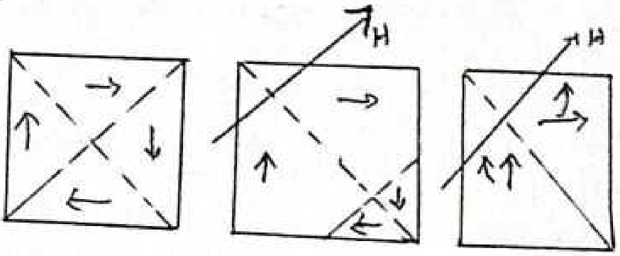
a) By the motion of domain walls

b) By rotation of domains.

a) By the motion of domain walls

When a small magnetic field is applied, the domains with magnetisation direction parallel or nearly parallel to the field

- Domain growth occurs due to the movement of domain walls away from the minimum energy state.



a) Random domain alignment

b) Domain wall movement

c) Domain rotation

b) By rotation of domains

- Magnetic field is increased to a large value further domain growth becomes impossible through domain wall movement.
- rotate as to be in complete alignment with the field direction

Types of Energy involved in the process of domain growth

i) Exchange energy
ii) Magnetostatic energy
iii) Crystal anisotropy energy
iv) Magnetostrictive energy.

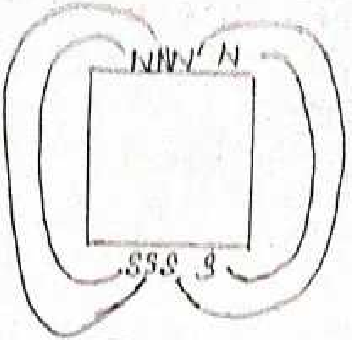
i) Exchange energy

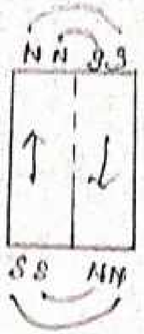
- Energy associated with the quantum mechanical coupling
- Align individual atomic dipoles within a single domain.
- Depends upon the inter atomic distance.
- Having a single domain structure

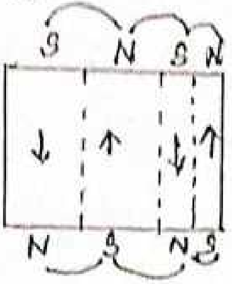
ii) Magnetostatic energy

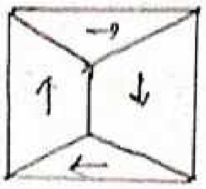
- Energy present in any ferromagnetic materials
- Due to the presence of resultant dipole moment in that material even in the absence of external magnetic field.

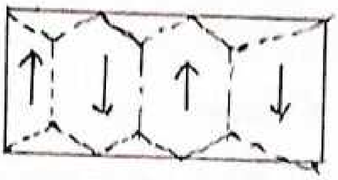
- Specimen can be reduced by dividing single domain into two domains





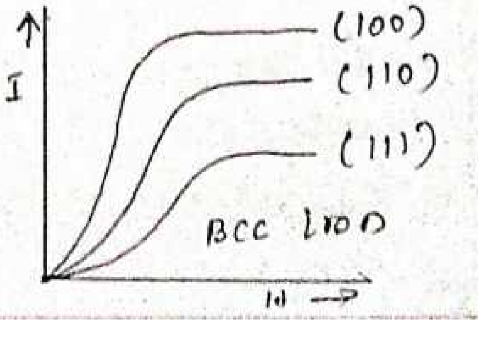






- N domains reduces the magnetic energy to 1/N of the magnetic energy of the material with single domain.
- Due to the introduction of triangular domains at the top and bottom of the crystal. These triangular domains are called closure domains.

iii) Crystal anisotropy energy



· Function of crystal orientation

· Different crystallographic directions have been drawn

· To produce magnetic saturation in [111] direction.

The difference in magnetic energy to produce saturation in an easy [100] direction and hard [111] direction is called crystal anisotropic energy.

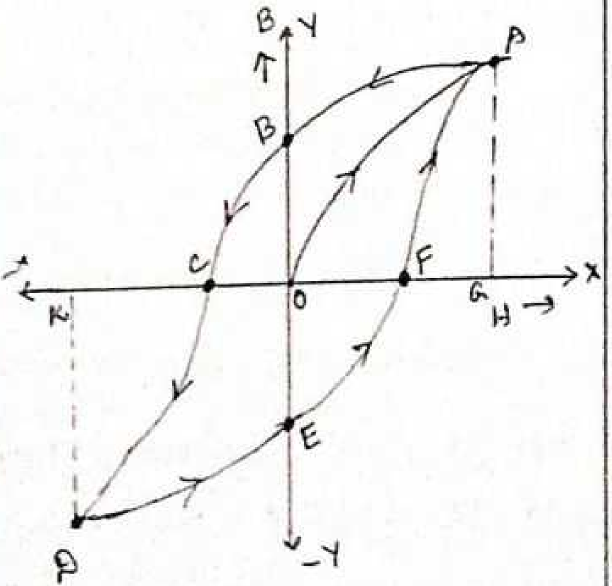
iv) Magnetostrictive energy

when a material is magnetised, it suffers a change in dimensions. This phenomenon is known as magnetostriction.

· Either expand or shrink.

· The work done by the magnetic field against elastic restoring forces is called the magneto elastic energy or magneto strictive energy.

④ Explain hysteresis - M versus H behaviour and give explanation of hysteresis on the basis of domain theory.



· The word hysteresis means lagging behind

· Intensity of magnetisation and magnetic induction lag behind the magnetising field.

· If the magnetising field is made zero, the value of I and B are not zero.

· There is a tendency in the material to retain its magnetic property. This lagging of I and B behind is is called hysteresis.

- The Variation of B with respect to H is represented by a closed loop or Curve and this loop is called hysteresis loop or Curve.

- The magnetic induction B increases along the curve OA with the magnetic field H.
- Beyond the point A, even when the magnetic field is increased, the magnetic induction does not increase and it remains constant.
- The specimen is Saturated with magnetisation.
- Now the value of magnetic field is decreased, but the magnetic induction does not decrease in the same rate as it increased.

- When H=0, B ≠ 0, the magnetic induction has a definite value represented by OB and it is known as retentivity.
- The applied magnetic field H is reversed and increased gradually till the point C is reached.
- The magnetic induction B becomes zero at the point C and it is known as Coercivity.

- Increase of magnetic field H, the magnetic induction increases along CD in the reverse direction as shown in the graph.
- If the magnetic field is Varied backwards, the magnetic induction follows a curve DEFA. This completes one cycle of magnetisation. The loop ABCDEFA is called hysteresis loop.

Explanation on the basis of domain theory

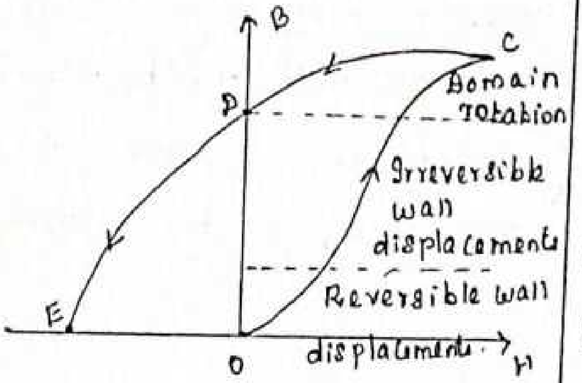
This is due to

i) Motion of domain walls

ii) Rotation of domain walls.

When a small external magnetic field is applied the domain walls are displaced slightly in the direction of magnetisation.

- when the applied magnetic field is removed, then the domain walls return to its original position and these domains are known as reversible domains.



- If the magnetic field is increased, the magnetization increases rapidly with H.
- Even when the magnetic field is removed, because of the displacement of domain wall to a very large distance, the domain boundaries do not come back to their original position. This process is indicated as AB. These domains are called irreversible domains.
- At point B all the domains got magnetised along the easy.
- The anisotropic energy is stored in the hard direction which is represented by BC
- The specimen is said to have attained the maximum magnetisation.
- After the removal of external magnetic field the material has maximum magnetisation called residual magnetism.
- The amount of energy spent to reduce the magnetisation to zero is called coercivity.

(K) Discuss about the types of magnetic materials.

Magnetic materials also classified into two types based on magnetisation

i) Soft magnetic materials

ii) Hard magnetic materials.

Soft magnetic materials

Definition :

Materials which are easy to magnetise and demagnetise are called soft magnetic materials.

Properties of soft magnetic materials

- Soft magnetic materials can be magnetised and demagnetised easily.
- High Permeability
- Low residual magnetism
- Low coercivity
- Low hysteresis loss
- Magnetic energy stored is Low.

Examples :

Cast iron, carbon steel, Silicon steel

Applications of soft magnetic materials.

Cast iron : used in the structure of electrical machinery and the frame work of D.C. machine

Carbon steel :

Has high mechanical strength is used in making motor of turbo alternators

Silicon steel

used for the construction of poles of motor and dynamo and core plates of transformer.

Hard magnetic materials

Definition

Materials which retain their magnetism and are difficult to demagnetise are called hard magnetic materials.

Properties :

- Low permeability
- Strong repel the magnetic field.
- High retentivity and Coercivity.
- High magnetising force to attain magnetic Saturation.
- Large hysteresis loop area
- Large energy loss.

Examples of Hard magnetic materials.

Tungsten steel

Cobalt steel

Alini

Alnico

Applications

Tungsten Steel

used in making permanent magnets for dynamos and motors.

- Cobalt Steel :

  used in motors, fans and heavy duty instruments

- Alini

  used in the design of Portable and light weight instruments.

- Alnico

  Used for the production of permanent magnets in Smaller Size.

Q ⑥ Explain magnetic Principle in data Storage

Magnetic materials are used for recording / reading of the audio and video signals. They are also used in storage devices such as magnetic tapes, floppy disks and hard disk.

Storage of Magnetic Data

- Storage capacity of the main memory of a computer system is limited.
- It is necessary to store many data on these memories.
- Additional memory called auxiliary memory is used in the most of the systems. It is known as secondary storage devices.
- Also known as back up storage.
- Used to store large volume of data on a permanent basis.

The main purpose of secondary storage is

a) To increase the memory capacity

b) To store the data permanently.

The most common secondary memories

i) Magnetic tapes

ii) magnetic disk

iii) Ferrite core memories

iv) Magnetic bubble memories.

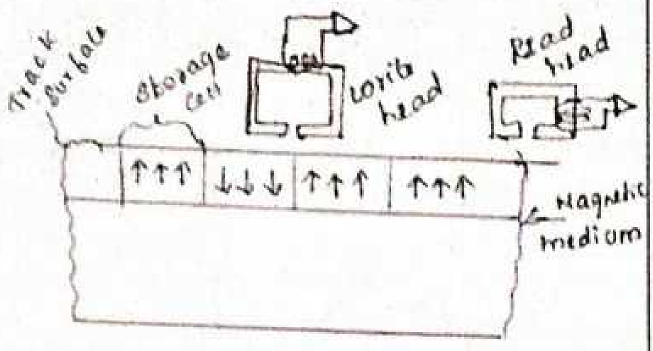
Magnetic tapes

- Most popular storage medium for data recording.
- Tapes are available in the market in form of a large reels or small cartridges or cassettes.

Recording and Reading Process

- Read / write head.

⑦



- A specially constructed electromagnet read/write head is used.
- The north and south pole of the read/write head is seperated by a narrow gap is called head gap.
- Data (0 and 1) in the form of electrical signal is applied to the write head
- stores data on magnetic tape as logic '1' in a storage cell with magnetisation in one direction.
- Next data is stored as logic '0' in the next storage cell with the magnetisation in the opposite direction.
- Data is stored on the tape in blocks.
- The data stored on the magnetic tape is read out by the read/write head, when the tapes moves across the head.
- the storage cell induces small electrical signal in the head.
- electrical signal gives information about the stored data.
- The digital information stored is read out.

Advantages

- Easy to handle and portable
- More compact and easier to handle.
- storing data on tapes is cheaper.

Disadvantages

- It is a sequential access memory and hence access time is more.

Application

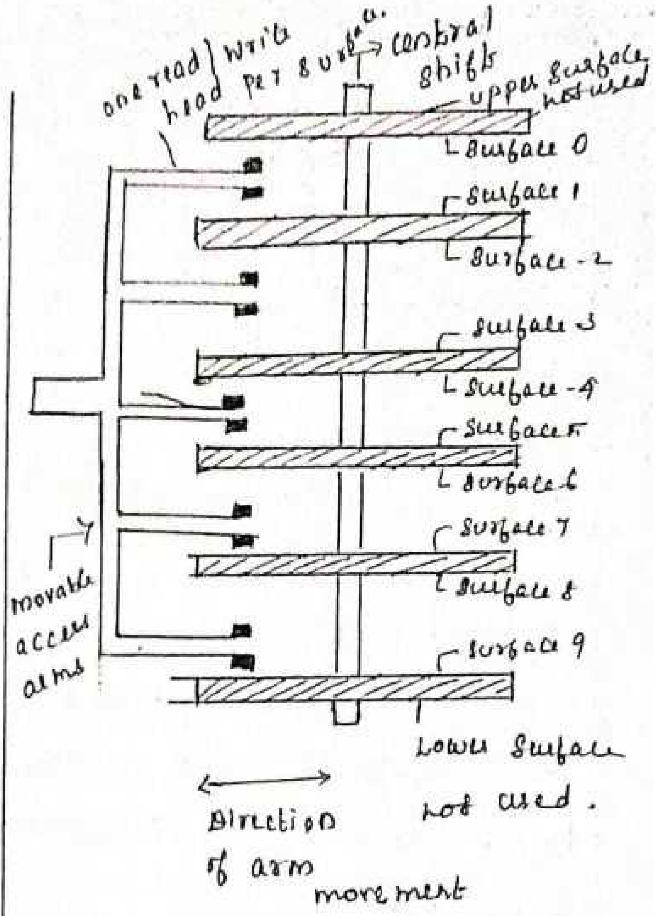
- used only for long-term storage and back up.
- Tapes are also used for transporting large amount of data.

Magnetic Hard disc

- Hard disk is used for storing a large amount of information.
- Hard disk is completely sealed and it is protected from the dust particles. This hard disk is also known as Winchester disk.

Construction

- It consists of a number of magnetic disks or aluminium platters.
- All platters are packed together and mounted on a common shaft.
- The central shaft rotates at the speed of 3600 or more revolutions per minute.
- hard disk move simultaneously in the same direction.
- The disk packs, read/write heads and the access mechanism are sealed in an airtight dust free container.
- A set of corresponding tracks in all sides is called a cylinder.
- The presence of a magnetized spot represents '1' bit and its absence represents '0' bit.



Advantages

- Very large storage capacity.
- More files can be permanently stored.
- Very high speed in retrieving data.

Disadvantages.

- Hard disks are not easily portable
- Cost is more
- More chance for errors.

Application

- Hard disk is a common secondary storage device for all types of computers.
- Storage capacity ranging from 20 Mega bytes to 4 giga bytes.

⑦ Write short notes on GMR (Giant Magneto Resistance)

Definition

The effect is observed as a significant change in the electrical resistance depending on whether the magnetization of adjacent ferromagnetic layers are in a parallel or an antiparallel alignment.

The overall resistance is relatively low for parallel alignment and relatively high for antiparallel alignment. the magnetization direction can be controlled. for example by applying an external magnetic field.

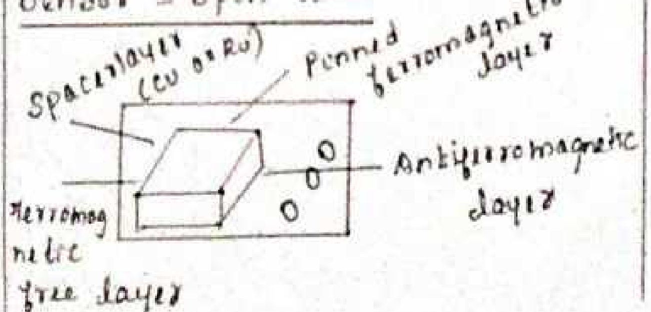
Two geometries are commonly used in GMR studies

a) current in plane of layers

b) current perpendicular to plane of layers.

Giant magneto Resistance

Sensor - Spin Value



- A device that works on the principle of the GMR is a Spin valve.
- Device is used in magnetic hard discs for high density data storage.
- 4 layers altogether in a Spin valve.
- Two ferromagnetic layers are Seperated by a thin Spacer layer.
- one ferromagnetic layer is pinned.
- Pinned layer adding a fourth layer, a strong antiferromagnet.
- other layer called the free layer, produced by the data bits.
- Permalloy is a ferromagnetic layers. This Structure is called the Spin valve.
- Generates a significant change in electrical resistance due to GMR effect.
- As the bit travels under the head, the resistance goes down, the electrons donot Scatter very much and the

current flow increases.

- As the bit moves on, the resistance increases the electrons are scattered ~~ore~~ and the current decreases.
- As the bit travels further from the head, the resistance peaks and the current decreases to its lowest point.
- As the resistance change is quiet large, even small data bits can generate quite large resistance changes, thus increasing the capacity to store data bits in the hard disc.

unit : IV

optical properties of Materials

classification of optical materials - Carrier generation and recombination processes - Absorption, emission and scattering of light in metals, insulators and semiconductors - Photo current in a P-N diode - Solar Cell - LED - Organic LED - Laser diodes - optical data storage techniques.

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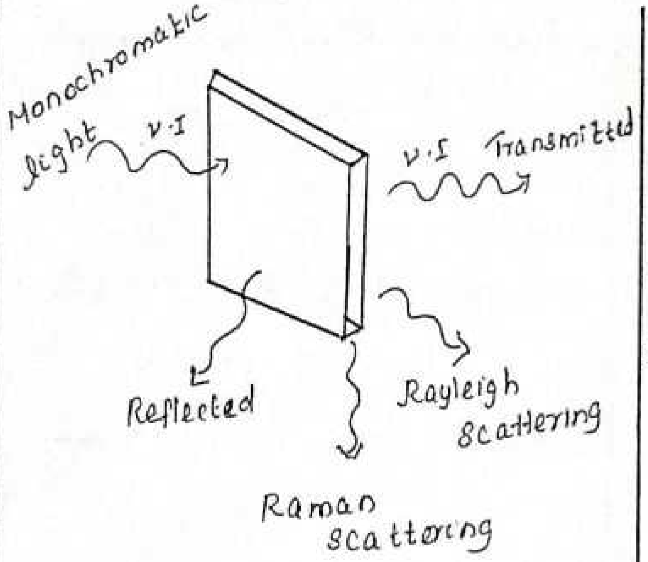
① Explain scattering of light in solids



- When a light beam incidents on a solid, the light radiation undergoes three processes

i) Scattered by the sample at various angles

ii) Absorbed by the sample

iii) Transmitted through the sample

- Light scattered in the opposite direction
- Light scattered in the same direction as the incident beam and recombining gives rise to refraction.
- forms of scattering such as Rayleigh scattering.
- Total incident flux of photons I_0,

$$I_0 = I_T + I_R + I_A \longrightarrow (1)$$

I_T - transmitted light intensity

I_R - reflected light intensity

I_A - absorbed light intensity.

Dividing I_0 on Both sides of eqn(1)

$$\frac{I_0}{I_0} = \frac{I_T}{I_0} + \frac{I_R}{I_0} + \frac{I_A}{I_0} \longrightarrow (2)$$

$$\boxed{1 = T + R + A}$$

T - fraction of light transmitted

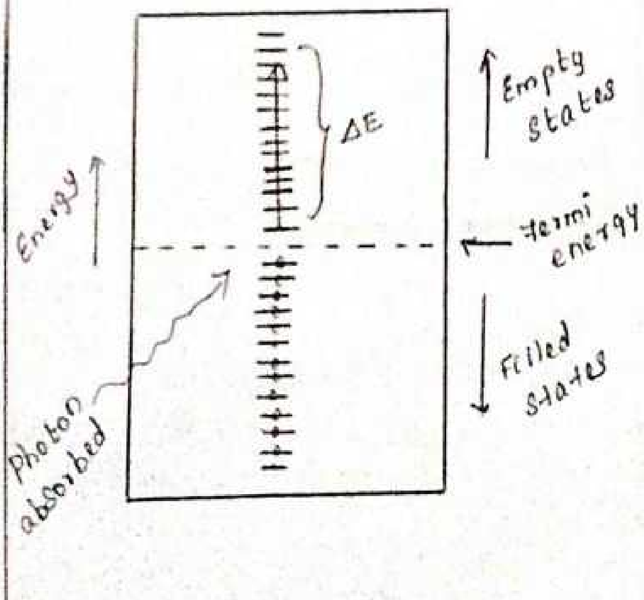
R - fraction of light reflected

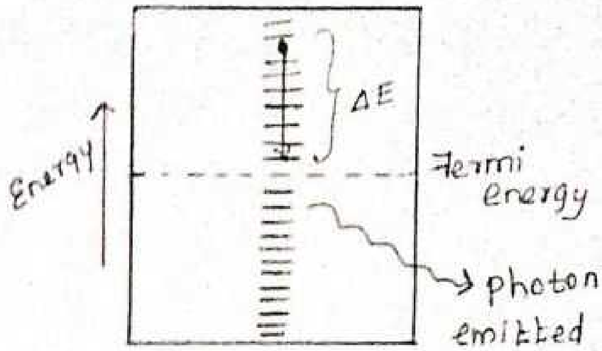
A - fraction of light absorbed.

② Describe absorption and emission of light in metal, Insulator and Semiconductor

① Absorption and Emission of light in metals :

- Metals are opaque.
- Incident radiation is absorbed.
- Total light absorption, is with in very thin outer layer, less than <u>0.1 μm</u>
- Metallic films thinner than 0.1 μm are capable of transmitting visible light.
- All frequencies of visible light are absorbed by metals.
- Continuously available empty electron states which Permit electron transitions.

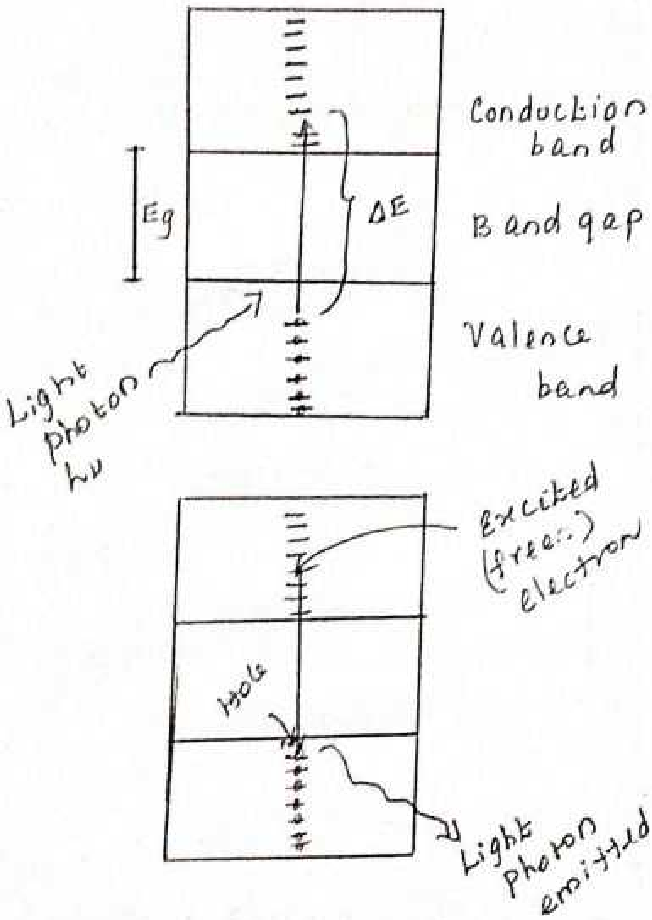




- Metals are <u>opaque</u> to all electromagnetic radiation on the lower end of the frequency Spectrum from radio waves, through infrared, visible and about to the middle of the ultraviolet radiation.
- Metals are <u>transparent</u> to high frequency x-rays and γ-rays.
- Most of the absorbed radiation is re-emitted from the surface in the form of visible light of the same wavelength appears as <u>reflected light</u>
- Reflectivity of most metals is between 0.90 and 0.95
- Colour of a metal is determined by the wavelength distribution of the <u>reflected radiation</u>
- Some metals when exposed to white light show a bright 'Silvery appearance'.

② Absorption and Emission of Light in insulators

- Absorption of a light photon may occur in an insulator.
- Excitation of an electron from valence band to conduction band after crossing the energy gap Eg.



- A free electron in the conduction band and a hole in the valence band are created.

$$\Delta E = h\nu \longrightarrow (1)$$

h – Planck's constant

ν – frequency of the light photon

Light photon absorption can take place only if

$$h\nu > E_g \longrightarrow (2)$$

$$\frac{hc}{\lambda} > E_g \longrightarrow 3 \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

c – velocity of light

λ – wavelength of the light photon

$$E_g(max) = \frac{hc}{\lambda\, min}$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}}{0.4\times10^{-6}}$$

$$= 4.96\times10^{-19}\,J$$

$$= \frac{4.96\times10^{-19}}{1.6\times10^{-19}} \qquad (1\,eV = 1.6\times10^{-19}\,J)$$

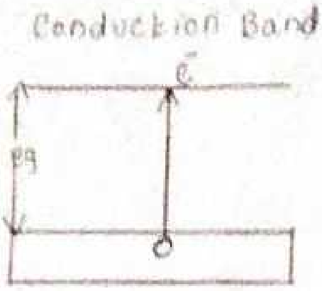
$$\boxed{E_g(max) = 3.1\,eV}$$

③ Absorption and emission of Light in Semiconductors

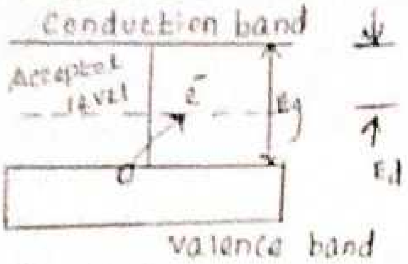
- In semiconductors, light photons is absorbed in several ways
- In intrinsic semiconductors such as Si, Ge and GaAs light photons is absorbed to create electron - hole pairs.
- Absorption cause electrons to jump across the energy

③

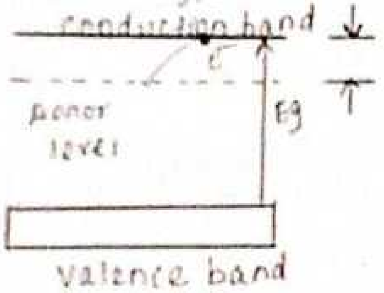
band gap from the valence band to the conduction band



a) Intrinsic



b) P-type



c) n-type

- Transition occurs

$$h\nu > E_g \rightarrow (1)$$

h - Planck's constant

ν - frequency of the light photon

$$\frac{hc}{\lambda} > E_g \qquad [\because \nu = c/\lambda]$$

λmax is about 0.7 μm

$$E_g(\text{min}) = \frac{hc}{\lambda_{max}}$$

$$E_g(\text{min}) = \frac{(6.62\times10^{-34})(3\times10^{8})}{0.7\times10^{-6}}$$

$$= 2.84\times10^{-19}\,J$$

$$= \frac{2.84\times10^{-19}}{1.6\times10^{-19}} \qquad (\because 1eV = 1.6\times10^{-19}J)$$

$$\boxed{E_g(\text{min}) = 1.8\,eV}$$

---

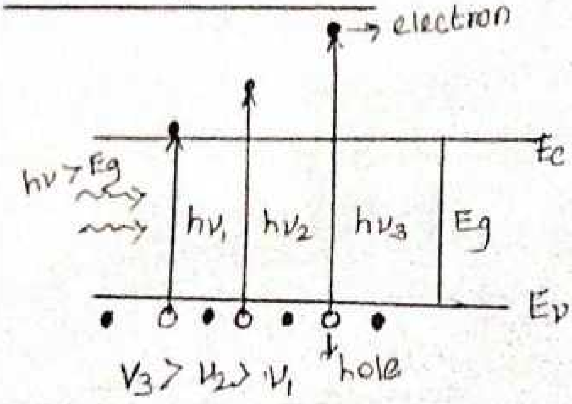
③ Explain Carrier Generation and Recombination Process

Carrier Generation

The Carrier generation is the Process where by electrons and holes are created.

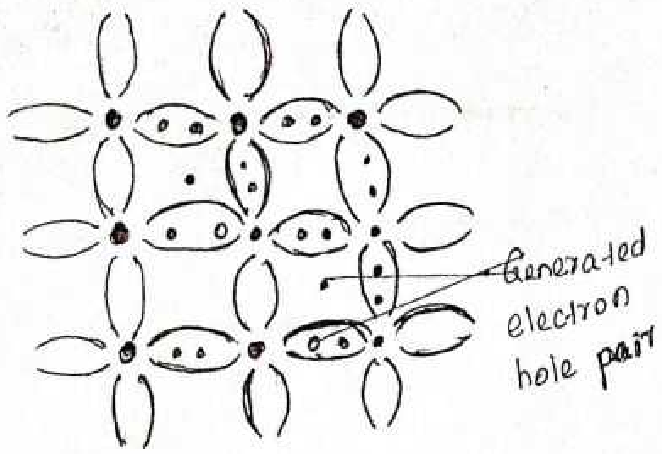
- Three types of carrier generations
  - i) Photogeneration
  - ii) Phonon generation
  - iii) Impact ionization

i) Photogeneration



- different wavelengths of light with different energies

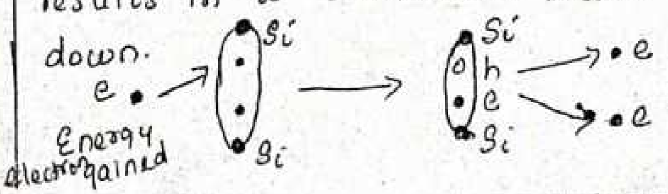
ii) Phonon Generation



- Phonon generation occurs when a semiconductor is under thermal excitation.
- with increase of temperature of the semiconductor, lattice vibrations increase gives more phonons.
- Due to more lattice vibrations, covalent bonds in the semiconductor break down and electron hole pairs are generated.

iii) Impact Ionization

- one energetic charge carrier will create another carrier
- Electrons gain energy from the applied electric field and hit other Si - atoms.
- a bond breaks out generating more carriers.
- very high electric field, it results in a avalanche break down.



Recombination Process

Recombination is the process where by electrons and holes are annihilated.

Recombination occurs in three ways

a) Radiative Recombination

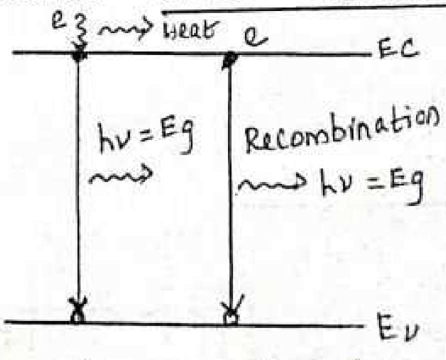
b) Shockley - Read - Hall Recombination

c) Auger Recombination.

a) Radiative Recombination

- It occurs for direct band semiconductors
- one photon of energy $h\nu\ (= E_g)$ is emitted.

from conduction band minimum it falls to valence band maximum emitting light of energy $h\nu = E_g$. It is also called direct recombination



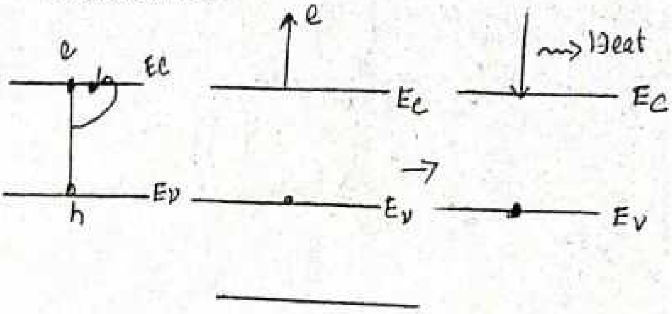
b) Shockley - Read Hall Recombination

Recombination process, electron from conduction band minimum come to a defect level intermediate between Ec and Ev by radiating energy as photons or phonons.

Auger Recombination

- Three carriers are involved
- Electron and a hole recombine
- Energy is given to the third free electron.

- Occurs heavily doped material.



④ Describe the construction and working of photodiode

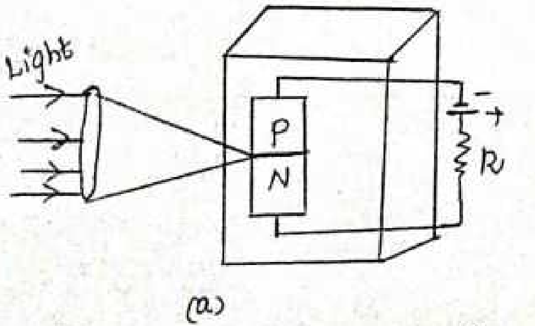
Photo diode : Definition

It is a reverse biased P-N junction diode which responds to light absorption

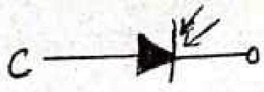
Principle

when light is incident on the depletion region of the reverse-biased Pn junction, the concentration of minority carriers increases. Therefore, reverse saturation current increases.

Construction



(a)



(b)

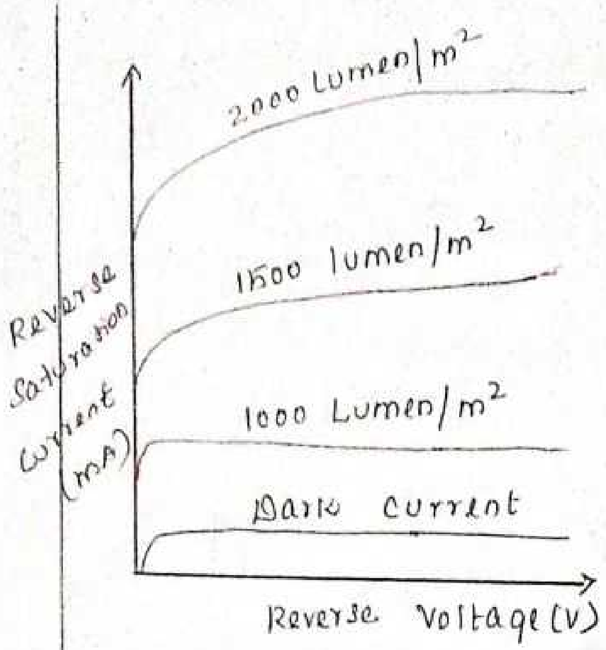
Working

- Photo diode is kept under dark condition
- Voltage is applied
- constant current independent of reverse bias voltage is obtained.
- Reverse saturated current is due to thermally generated minority charge carriers.
- It is called as dark current.
- electron hole pairs are generated.

Total reverse current is given by

$$\boxed{I = I_s + I_d}$$

I_s - short circuit current



The volt ampere characteristic curve is shown above.

i) The current increases with increase in the level of illumination for a given reverse voltage.

ii) only for the dark current at zero voltage the current is zero.

Applications

- Light detection system
- Reading of sound track in film
- Light operated switches
- High-speed reading of computer punched cards and tapes
- Used to switch on the current at a very fast rate.

---

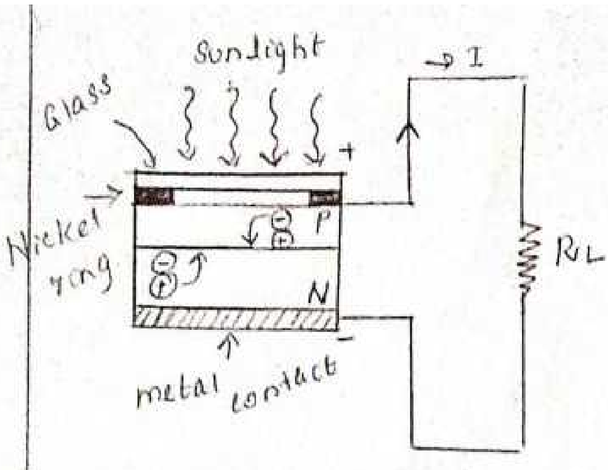
⑤ Explain the construction and working of a solar cell

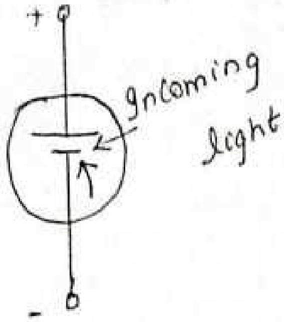
It is a P-N junction diode which converts solar energy (light energy) into electrical energy.

- Common materials for solar cells silicon (Si), Gallium Arsenide (GaAs), Indium Arsendide (InAs)
- The most common is silicon.

Construction

- Consists of PN junction diode made of silicon
- P-N diode made with glass window on top
- Light fall on P and N type materials.
- Thickness of the P-region is kept very small.
- Recombination takes place.

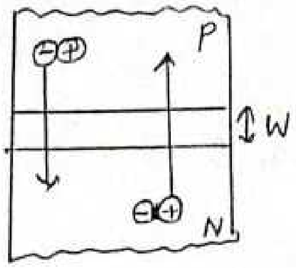


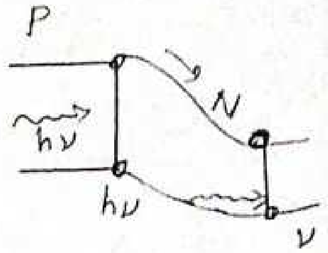


- Thickness of N-region is also kept small to allow holes generated near the surface to diffuse to the junction before they recombine.
- A nickel ring is provided around the P-layer which acts as the positive output terminal.
- A metal contact at the bottom serves as the negative output terminal.

Working

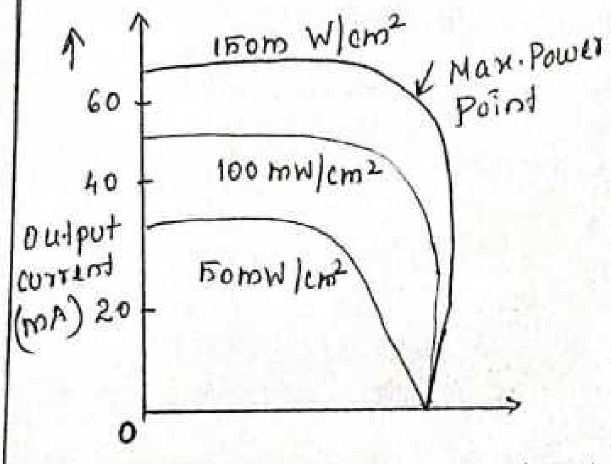
- Light falls on P-N junction diode, photon energy is sufficient to break the covalent bond and produce electron hole pair.
- Electron hole pairs are generated in both P and N sides of the junction.
- Electrons and holes reach the depletion region by diffusion.





- Minority carrier electrons in the P-side cross the barrier potential to reach N-side and the holes in N-side move to the P-side.
- Electrons and holes are accumulated on the two sides of the junction.

V.I characteristics



The maximum power output is obtained when the solar cell is opened at the knee of the curve

⑧

Advantage

- The Solar cell operates with fair efficiency
- It has unlimited life
- It can be mass produced
- It has a high power capacity per weight.
- Its size is small and compact.

Disadvantages :

Solar energy is not available round the clock. It cannot be obtained during night time.

Uses :

- used to provide commercial electricity.
- used to give Power to the calculators and watches.
- used in satellites and space vehicles.

---

⑥ Explain the construction and working of a LED with energy band diagram

It is a p-n junction diode which emits light when it is forward biased.

Principle

The injection of electrons into the P-region from n-region makes a direct transition from the conduction band to valence band.

- electrons recombine with holes and emits photons of energy Eg.

The forbidden energy gap is given by

$$\boxed{E_g = h\nu}$$

h - Planck's constant

ν - frequency of the emitted radiation

$$\nu = c/\lambda$$

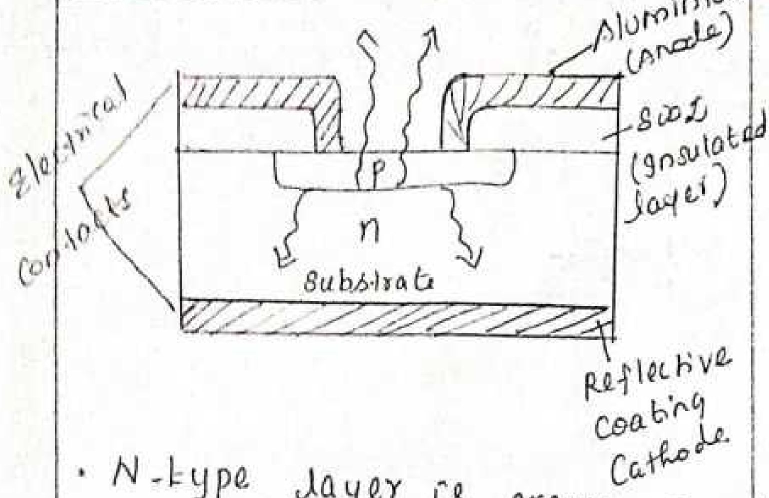
c - Velocity of the light

λ - wavelength of the light

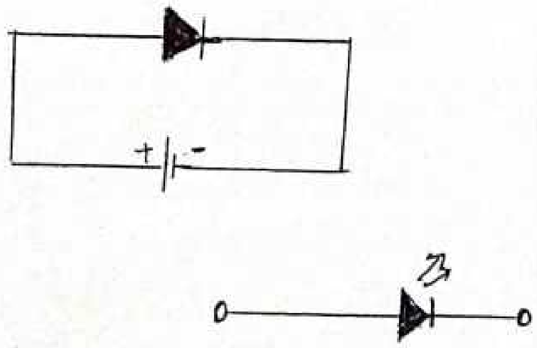
$$E_g = \frac{hc}{\lambda}$$

$$\boxed{\lambda = \frac{hc}{E_g}}$$

Construction



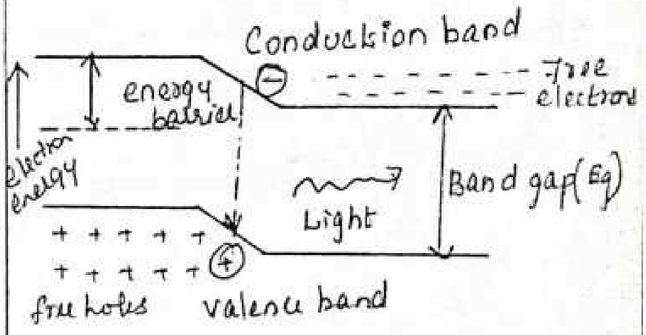
- N-type layer is grown on a substrate
- P-type layer is deposited on it by diffusion.
- Recombination takes place.
- For maximum light emission a metal film anode is deposited at the outer edges of the P-type layer.
- The bottom of the substrate is coated with a metal (gold) film.
- Provides cathode connection.



Circuit and symbol of LED

Working

- When P-n junction diode is forward biased, the barrier width is reduced
- Free electrons and holes have sufficient energy to move into the junction region.
- Light radiation from LED is caused by the recombination of holes and electrons



Advantages of LEDS

- LED's can be turned ON and OFF in less than 1 nano second. So they are known as fast devices.
- Light modulation can be achieved with pulse supply.
- Long life time
- It has low drive voltage and low noise.
- It is easily interfaced to digital logic circuits.

⑩

- variety of LEDs are available which emit light in different colours like green, red yellow etc.

Disadvantages of LEDs

- Require high Power
- Preparation cost is high when compared to LCD

Applications and uses of LEDs

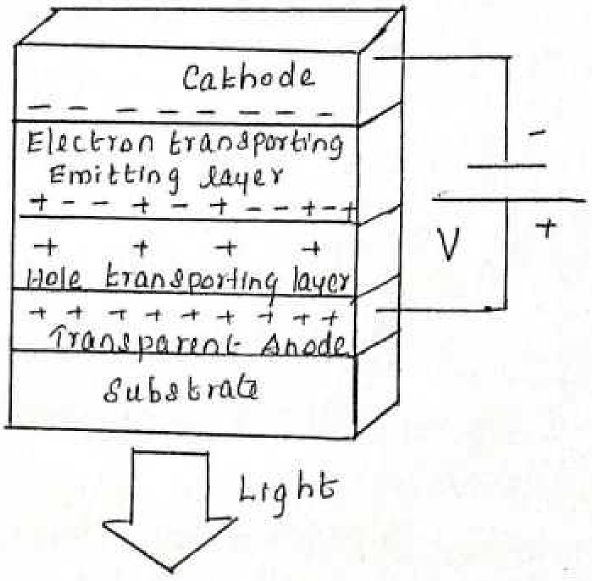
- used in numeric and alphanumeric display devices.
- used as indicator lamps.
- used as light sources in fiber optic communication system.
- used in burglar alarms.
- used for picture phone.
- used as pilot light
- used with photo diodes or photo transistors to enable short range wireless communication.

---

⑦ **What is OLED? Explain the basic concept of OLED, types, advantages, disadvantages and application.**

Organic light emitting diodes are solid state devices made up of thin films of organic molecules that produce light with the application of electricity.

- Layer usually combines a polymer substance that allows suitable organic compound to be deposited.
- Emit light of different colours.
- The OLED consists of an emissive layer, a conductive layer, a substrate anode and cathode terminals.



- Layers are made of special organic polymer molecules that conduct electricity.
- Conductivity range is between insulators and conductors. So they are called organic semi conductors.

Working

- An organic film is contacted by a metal electrodes on both sides.
- when a voltage is applied, positive charges are injected into the organic material. from one contact.
- The negative charges are injected from the other side into emissive layer.
- When two different charge carriers meet, they recombine each other produce energy in the form of light photon.

Types of OLEDS

- There are five types of OLEDS.

i) PLED
ii) POLED
iii) TOLED
iv) SOLED
v) IOLED

i) PLED

Polymer Light-Emitting Diodes (PLED) involve an electroluminescent conductive polymer that emits light when it is subjected to an electric current.

ii) POLED

Patternable Organic Light-Emitting device uses a light or heat activated electroactive layer.

iii) TOLED

Transparent Organic Light Emitting Device uses a transparent contact to create displays.

iv) SOLED

Stacked OLED uses a novel pixel architecture that is based on stacking the red, green and blue subpixels on top of one another.

v) IOLED

Inverted OLED uses a bottom cathode that can be connected to the drain end of n-channel TFT.

Advantages

- **Robust Design** : to use in portable devices such as cellular phones, digital video cameras, DVD players, car audio equipment etc.
- **Viewing Angles**

can be viewed upto 160 degrees

• High Resolution :

High Information applications including videos and graphics.

• Electronic paper :

OLEDs are paper-thin

• Production Advantages

Upto 20% to 50% cheaper than LCD processes.

• Video capabilities

They hold the ability to handle streamlined video, the display and cellular phone market.

• Power usage

Takes less power

Drawbacks

- OLEDs is the limited life time of the organic materials.
- Color - The reliability of the OLED is still not upto the mark.

After a month of use, the screen becomes non-uniform.

Applications

- Used in commercial applications such as mobile phones and portable digital audio players, car radios, digital cameras.
- used in television screens, computer displays, advertising information and indication.

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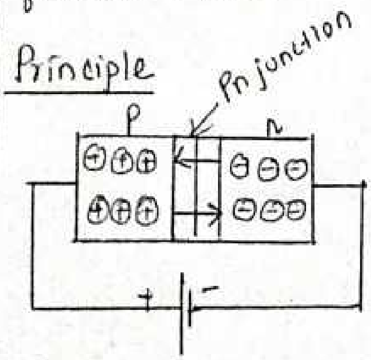
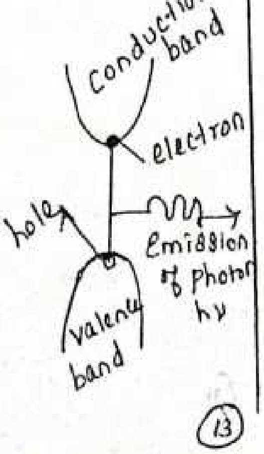
⑧ **Describe the construction and working of laser diodes. What are the advantages of these diodes?**

- It is a specially fabricated P-n junction diode. This diode emits laser light when it is forward-biased.

Principle



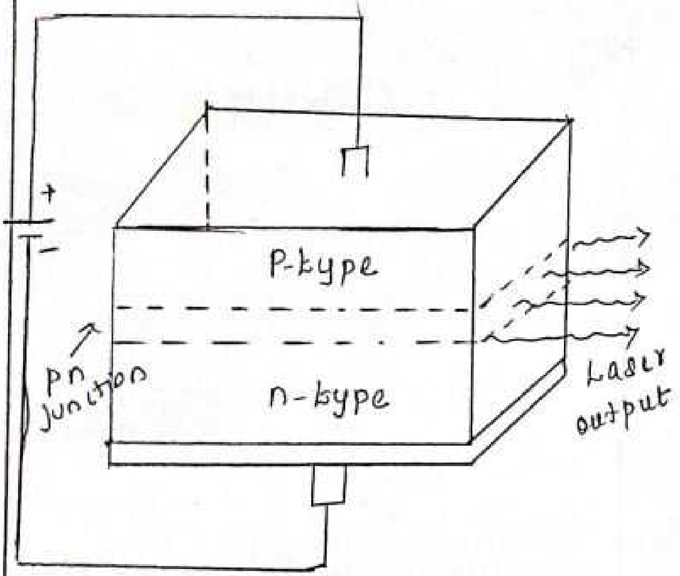


- When the P-n junction diode is forward biased, the electrons from n region and holes from P-region cross the junction and recombine with each other.
- During the recombination process, the light radiation is released from a direct band gap semiconductors like GaAs

The light radiation is known as recombination radiation.

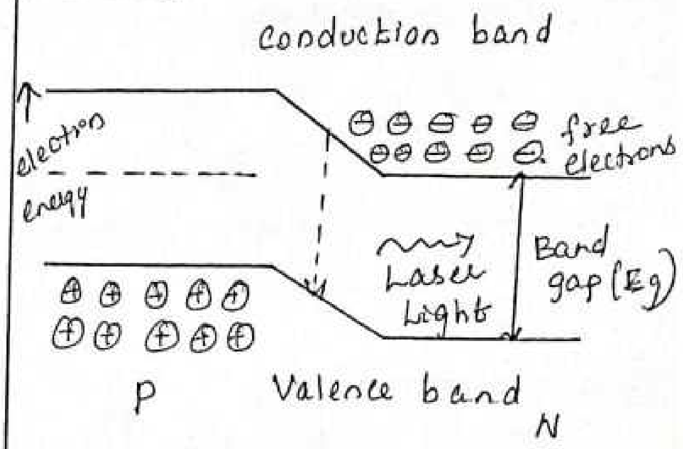
- Stimulated emission takes place and laser light is produced.

Construction



- The active medium is a p-n junction diode made from a single crystal of gallium arsenide.
- Crystal is cut in the form of a platelet having a thickness of 0.5mm.
- Platelets consists of two regions n-type and P-type
- The metal electrodes are connected to both upper and lower surfaces of the semi-conductor diode.
- The forward bias voltage is applied through metal electrodes.
- Photon emission is stimulated.
- The end faces of the Pn junction are well polished and parallel to each other.
- Act as an optical resonator

Working



- When Pn junction is forward biased the electrons and holes are injected into junction region.
- Region around junction contains a large number of electrons in the conduction band and holes in the Valence band,
- Electrons and holes recombine with each other.
- During recombination, light photons are produced.
- When the forward-biased voltage is increased, more light photons are emitted.

- Photons trigger a chain of stimulated recombinations
- Photons moving at the plane of the junction travel back and forth by reflection between two polished surfaces of the junction.
- The light photons grow in strength
- After gaining enough strength laser beam of wavelength 8400 Å is emitted.

The wavelength of laser light is given by

$$E_g = h\nu = hc/\lambda$$

$$\boxed{\lambda = \frac{hc}{E_g}} \qquad (\because \nu = c/\lambda)$$

E_g - band gap energy in joule

Characteristics

- **Type** : Solid State Semiconductor laser
- **Active medium** : A single Crystal of gallium arsenide
- **Pumping method** : Direct Conversion method
- **Power output** : a few mW
- **Nature of output** : Continuous wave or pulsed output
- **Wavelength of output** : 8200 Å to 8500 Å

Advantages

- Laser is very small in size and compact
- High efficiency
- It requires very little additional equipment
- It emits a continuous wave output or pulsed output

Disadvantages

- Large divergence
- Purity and monochromaticit are poor.
- Poor Coherence and Stability

Applications of Laser diode

- Used in fibre optic Communication.
- Used in various measuring devices such as range finders, bar-code readers.
- Used in printing industry

---

⑨ Explain optical data Storage techniques and different types of optical disc

- Techniques have resulted in increased storage capacities.
- classified into Surface storage and Volume storage.

Surface Storage

Optical tape

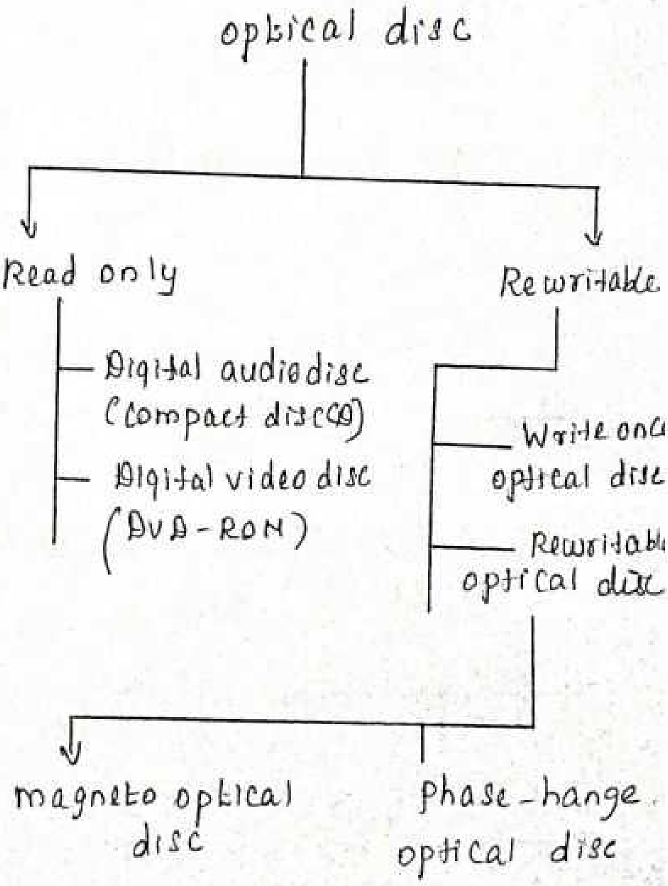
- Optical tapes for recording optical information.
- Examples : Cine movie film rolls
- Acoustical informations are recorded in such tapes as Sound tracks.

optical disc (compact Disc)

- The principle of optical disc is that the data to be stored is first converted into binary form as 0s and 1s.
- During the read-out process, variation in the reflected intensity of laser is converted back to data.

classification of optical disc

- Two types
  i) Reading only
  ii) Recording and reading type
- optical disc make use of laser diode, lenses and photodiodes.
- During recording it changes electrical information into optical information and then records the information on the optical disc.



CD audio

- Substrate of the disc is either plastic or photo-polymer
- First audio signal to be stored is converted into binary.
- injection molding
- CD at low cost is possible
- Audio songs are available in CD.

CD ROM

- Video signal is converted into binary and stored in a metallic master.
- Read any number of times
- Cannot be changed

CD WORM (write once Read many)

- Chemical coating on the substrate.
- Creates reflecting and non-reflecting micro-points
- Write the data once
- Read number of times
- data written cannot be either copied or erased
- Rewritten
- Write once and Read many (WORM)
- For copies each disc has to be written burning the chemical coating.
- Use to record functions such as marriages etc.

CD R/W (Read/Write)

- write the data, read and rewrite after erasure.
- Phase change materials and magneto-optic materials are used in general.

DVD Digital Versatile Disc

- Read only optical system called Digital Versatile.
- Enough capacity
- Compressed video

Digital Video Recording (DVR)

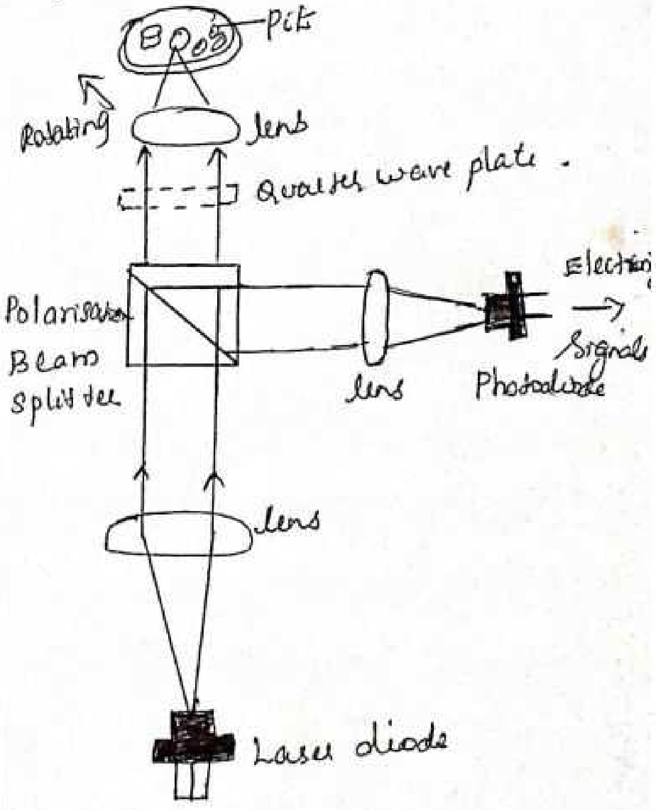
- 22 GB can be recorded on a single layer of 12cm disc.
- recording of high definition digital video.
- By reducing the spot size using a laser of shorter wavelength
- objective lens of higher numerica aperture a real density is increased.

Advantages of optical disc

- optical discs have several advantages over semiconductor memories.
- Larger storage capacity.
- Shorter access time size
- used in terminal equipment of computers.

Read only optical Disc Equipment

- Reading data from digital audio disc.
- Compact discs which are 120 mm in diameter are typical digital audio discs.
- It includes the read only memory for data memory
- interactive compact disk for multimedia use.
- Audio information is digitally recorded in stereo on the surface of a CD in the form of microscope 'pits' and 'flats'.
- Light passes through the lens and focussed on the surface of a disk.
- CD rotates, the lens and beam follow the track under control of a servo motor.
- The recorded track is reflected back from the track through the lens and optical system to infrared photodiodes
- The signal from the photodiode in then used to reproduce the digitally recorded sound.



unit : IV

Nanodevices and Quantum Computing

Introduction - quantum Confinement - quantum Structures: quantum wells, wires and dots - band gap of nano materials. Tunneling - Single electron phenomena : Coulomb blockade - resonant - tunneling diode - Single electron transistor - quantum cellular automata - Quantum System for information processing - quantum States - classical bits - quantum bits or qubits - CNOT gate - multiple qubits - Bloch Sphere - quantum gates - advantage of quantum Computing over classical Computing.

---

① Discuss Quantum Confinement and Quantum Structures

Quantum confinement

Definition

It is a process of reduction of the size of the Solid such that the energy levels inside become discrete

- Small droplets of isolated electrons are created.
- The energy of a small volume of such materials are quantized.
- Have tunable electrical properties.
- when the size of the particle is too small to be comparable to the wavelength of the electron.
- Free to move during confinement.
- In bottom-up approach, low volume Structures are built atom by atom.
- In top-down approach material is removed from

one or more three dimensions.

Quantum Structures

Definition :

when a bulk material is reduced in its size, atleast one of its dimension, in the order of few nanometres, then the structure is known as quantum structure.

A structure in which the motion of the electrons or holes are confined in one or more directions by potential barriers is called quantum confined structure.

· Three types based on the confinement directions.

i) quantum well
ii) quantum wire
iii) quantum dot

i) quantum well (2-dimension)

Definition :

when the electrons are confined inside a region of minimal width, if one dimension is reduced to the nanometre range while the other two dimensions remain large, we get a structure known as quantum well.

· The particles are confined in one dimension hence, they are considered to be quantum confinement.

use :

Quantum wells are now widely used to make semiconductor lasers and other important devices

ii) Quantum wire

when the electrons are confined in two mutually perpendicular directions, then the structure is known as quantum wire.

If two dimensions are reduced and one remains large, the resulting structure is quantum wire.

Examples

nanowires, nanorod. and nanotube

iii) Quantum Dots (0- dimension)

Definition

when all the three dimensions are minimized the resulting structure is known as quantum dot

- only confined states.
- No moving carriers.

Use :

used in a quantum computer and lasers

- the electrons are free to move and these structures exhibit "partial confinement"
- quantum dots exhibit "total confinement".

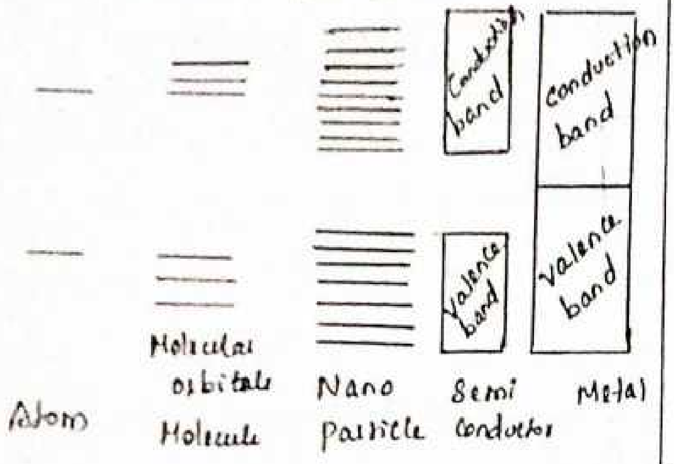
---

② Discuss notes on Bangap of Nanomaterials, Tunneling - Single electron phenomena, Coulomb- Blockade effects, Quantum size effect

i) Band gap of Nanomaterials

- Metals and semiconductors are determined by their electronic band structure.
- Band structure changes Particle size.
- Lies between the discrete density of states as in atoms and molecules.
- As the size of the material decreases, the energy Seperation between the adjacent levels increases.
- The particles that show this size quantization effect are called Q - Particles or quantum dots.
- Increasing spacing levels.

- Band gap already exists in the bulk state
- As the size of metal nano particles decreases, they tend to lose their metallic character and become semiconductors.



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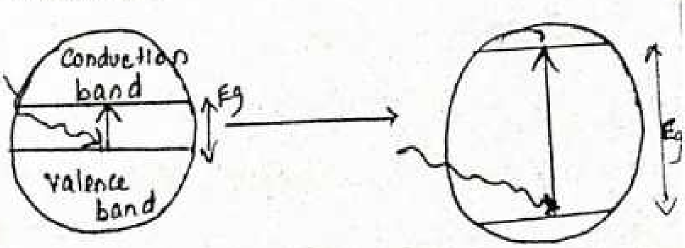
ii) Quantum Size Effect

- when the size of a nonocrystal becomes smaller than the debroglie wavelength, electrons and holes get spatially confined
- electrical depoles get generated.
- The discrete energy levels are formed.
- As the size of the material decreases, the energy seperation between adjacent levels increases.
- Quantum size effect is most significant for semiconductor nanoparticles.

- Band gap energy is the order of a few electron volts.
- when photons of light fall on a semiconductor, the photons are absorbed.
- A sudden rise in absorption is observed when the photon energy is equal to the band gap.
- As the size of the particle decreases, absorption shifts towards the shorter wavelength.
- A change in absorption causes a change in the colour of the semiconductor nanoparticles.

For example, bulk cadmium sulfide is orange in colour and has a bandgap of 2.42eV.

- It becomes yellow and white as its particle size decreases and the bandgap increases.



iii) Tunneling - Single Electron Phenomena

- The phenomenon of penetration of charge carriers directly through the potential barrier instead of climbing over it is called tunneling.
- Transistor is the most important device.
- turning ON and OFF making logic decisions.
- Microchips have over a billion transistors, each one turning ON and OFF a billion times every second.
- more transistors are squeezed into the same amount of semiconductor space.
- quantum effects will play a significant role.
- In 1970, to switch ON a silicon transistor required about 10 million electrons.
- Present day transistors require closer to 10,000 electrons.
- Very well be practical
- The single electron devices are sensitive to the transfer of even single electron change.
- High speed operation with lower power dissipation.

iv) Coulomb Blockade Effects

- As the size of the quantum dot decreases, the charging energy W_c of a single excess charge on the dot increases

Definition

The charging effect which blocks the injection or rejection of a single charge into or from a quantum dot is called Coulomb blockade effect.

Condition for Coulomb blockade

- Coulomb forces upon each other.
- If two charges are the same kind, the force is repulsive.
- Condition for Coulomb blockade effect is

$$\boxed{W_c = \frac{e^2}{2C} >> kT}$$

C - capacitance of the quantum dot.

T - Temperature of the system

W_c - charging energy

(5)

③ Describe Construction and Working of Single Electron Transistor.

Definition

SET is three-terminal switching device which can transfer electrons from source to drain one by one.

Construction and Working

- It has a similar structure of a conventional Field Effect transistor (FET)
- Has tunneling junctions in place of P-n junctions
- quantum dot in the place of channel region FET.
- To control tunneling a voltage bias to the gate electrode is applied.
- seperate voltage bias supplied to source and drain electrodes

The energy E needed to move a charge Q, across a potential difference V is given by

$$E = VQ$$

$$V = \frac{E}{e} = \frac{W_c}{e}$$

$$E = W_c'$$

W_c - charging energy

$$V = \frac{\frac{e^2}{2C}}{e} = \frac{e}{2C} \quad \left(\because W = \frac{e^2}{2C}\right) \longrightarrow (1)$$

- electron can tunnel through Coulomb blockade of the quantum dot.
- At the proper gate voltage $V = \frac{e}{2C}$ potential energy of the dot is low enough to allow an electron to tunnel through Coulomb blockade energy barrier to the quantum dot.
- potential energy rises.
- electron tunnels through the Coulomb blockade.
- Dot is empty and potential is lower again, the process repeats.
- gate voltage V_g is zero, no current flows.
- First gate voltage is large enough to move an electron through the Coulomb blockade is called Vcoulomb.

$V_g = V_{coulomb}$

- $V_{coulomb} + \frac{e}{2C}$, two electrons can be moved on the quantum dot

- $V_{coulomb} + \frac{e}{2C} + \frac{e}{2C} =$

$$V_{coulomb} + \frac{e}{C'}$$

- Three electrons be moved on the quantum dot.
- ON and OFF states make an effective switch out of a SET.

Advantages

- Fast information transfer speed between cells via electrostatic interactions
- No wire is needed between arrays.
- used for the next generation quantum computer.

Limitations

- Very hard to fabricate by traditional optical lithography and semiconductor process

Applications

- used for mass data storage
- used in highly sensitive electrometer.
- SET is a suitable measurement set-up for single electron spectroscopy.

---

(4) Write notes on Resonant Tunneling Diode

- Device has two tunneling junctions.
- I-V characteristics shows negative differential resistance characteristic.

Definition

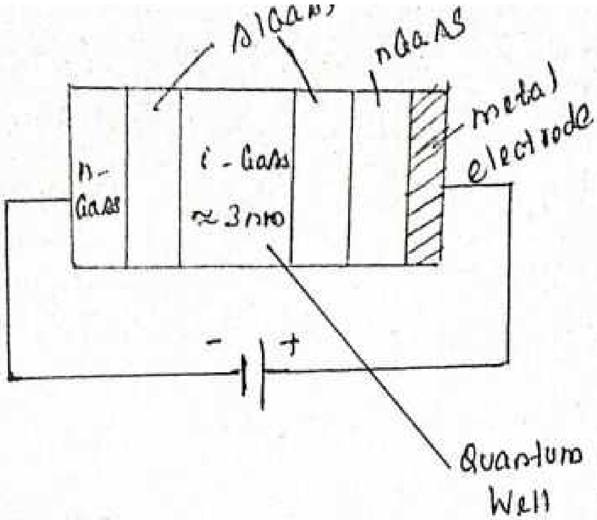
A resonant tunneling diode is a diode with resonant tunneling structure. The electrons can tunnel through some resonant states at certain energy levels.

Principle

When electron incident with energy equal to energy level of a potential well of thin barrier, then the tunneling reaches its maximum value. This is known as resonant tunneling.

Structure of RTD

- n-type using GaAs

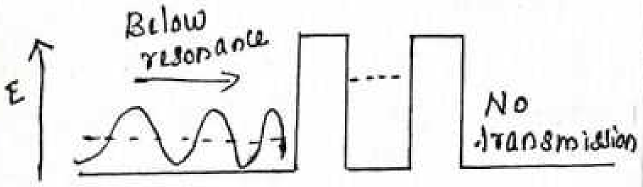


Working

Tunneling Control

Tunneling is controlled by applying a bias voltage across the device.

Without applied bias



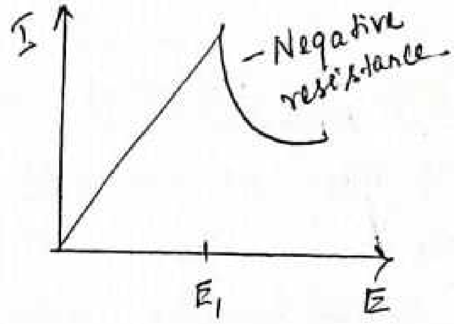
- Practically it is very difficult to control the barrier height
- energy matching
- achieved by biasing the potential barriers.

With applied bias

- when voltage is applied, the band diagram shifts.
- The voltage is varied untill the quantized discrete energy level corresponding to the potential well matches with the energy of the electron wave resonans tunneling occurs.

Current - Energy characteristic for a resonant tunneling diode

- transmission is low.
- As E tends to E_n, transmission will increase, becoming a maximum $E = E_n$
- As E increases, tunneling will increase, reaching a peak $E \neq E_1$
- E will decreasing current



- Decrease of current with an increase of bias is called negative resistance.

Application and uses of RTD

- Switching devices operate at tetra hertz frequencies.
- RTDs are very good rectifiers.

- used in digital logic circuits
- used in inverters, memory cells and transistors.

Advantages

- RTD are very compact.

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⑤ Write notes on Cellular Automata

- Microelectronics is based on scaling towards smaller structures.
- A cellular automaton is a d-dimensional lattice of bits that updates over discrete timesteps.

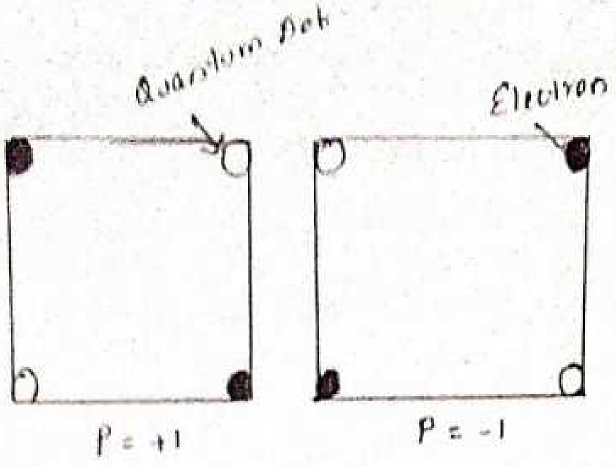
The CAs have many practical applications which include traffic models, fluid flows, biological pattern formation and reaction diffusion systems.

Quantum Cellular Automata (QCA)

- Emerging nanotechnology.
- CMOS (Complementary Metal oxide Semiconductor) technology has a lot of limitations while scaling into a nano level.
- New nano technology approach should be taken into account.
- QCA circuit is a majority gate.
- using Majority and inverter gates.
- Designed in a different manner from that of Boolean logic.

Explanation and Analysis

- technology is built up in cells.
- tunnel between the islands
- Due to coulomb repulsion they will always settle to one of the two stable states.
- binary "1", the other "0"
- No current flows into or out of the cell.



- Fundamental logic gate is the Majority Gate.
- 3 inputs
- Assuming three inputs labeled A, B and C, the logic function of majority gate is

$$\boxed{M(A, B, C) = AB + BC + AC}$$

Logical AND and OR functions can be implemented from majority input by presetting one input immutably to binary values 0 and 1.

QCA Clocking

- Controlling the potential barriers between adjacent quantum dots.
- no definite cell polarization.
- Raising the potential barrier decreases the tunneling rate, the electron begin to localize.
- when the potential barrier has reached its highest point, the cell is said to be latched.

Advantages of QCA

- It is "edge driven", meaning an input is brought to an edge of a QCA block.
- No power lines need be routed internally.
- Low power system because there is no current flowing.
- QCA cells are very small.

⑧ Discuss about Quantum System for information Processing

The quantum phenomena can be applied to quantum Computing, quantum information science, quantum communication and quantum metrology.

Quantum Information science

- The methods of encoding the information in a quantum system.
- Statics of quantum mechanics with their limitations.
- Provides core for all other applications such as computing communications, networking, sensing and metrology.

Quantum communication and networking

- Explains exchange of information by encoding into a quantum cryptography.
- Subset of quantum Communication.
- quantum properties help to design the secure communication system.

Quantum sensing and metrology

- used to measure important physical properties.
- Quantum sensors are based on qubits.

Quantum Computing

- quantum mechanical properties of superposition, entanglement and interfactions to enact computations.

fundamental concept to construct quantum computers

a) Harmonic oscillator quantum Computer

- Discrete eigen states are represented as $|n\rangle$, where $n = 0, 1, 2, \ldots \infty$. these represent qubits.
- Life time of qubits depend on the quality factor.
- A single quantum harmonic oscillator will have 2^n energy states in Hilbert space.

b) optical photon quantum Computer

- Photon can represent a quantum bit.
- Interact using non-linear optical media.
- Photons are detected with photo detectors.

c) optical cavity quantum electrodynamic computer.

- Coupling of single atom to a few optical modes takes place.
- It is done by placing single atom in optical cavities of very high Q.
- Fabry perot cavity containing a few atoms, optical field is coupled.
- Single photon can be good carriers of quantum information.

d) Iron trap quantum Computer

- Number of charged atoms are isolated and trapped.
- Atoms are cooled, kinetic energy is much less than their spin energy.
- Transitions can be made to perform quantum computation.
- Main components of the iron trap quantum computers are the electromagnetic trap with lasers, photo detectors and ions.

e) Nuclear magnetic resonance Computer.

- Based on the spins of atomic nucleus.
- Spin-Spin coupling can be large and controllable.
- Applied spins in a strong magnetic field.

Characteristics of Quantum Computers.

Superposition

- In classical computing, a bit has to be in a single state either 0 or 1
- In quantum computing a qubit can be in both ~~states~~ 0 and 1 simultaneously called superposition.

Entanglement

- Quantum Computing is Entanglement.
- Interacted at some point are entangled in pairs.
- If the state of one qubit in this pair is known then that of other can be determined.
- "Spooky action at a distance" by Eistein.

Working of quantum Computers

- quantum System uses quantum bits or qubits as the smallest discrete units to represent information.
- The property of quantum mechanics comes into play in qubit.
- The Superposition is removed and one of the two distinct states is returned based on the probabilities of each state.
- 2^n states are achieved which exist as combinations of os and is in parallel.
- The next step is to process information
- Shor's algorithm and Grover's algorithm function using the principles of quantum mechanics of Super position, entanglement and measurement.

Architecture of quantum Computer.

- Seen as blue print
- Architecture of the quantum Computer is a combination of classical and quantum parts.
- Divided into 5 layers, represented as functional Part of the Computer.

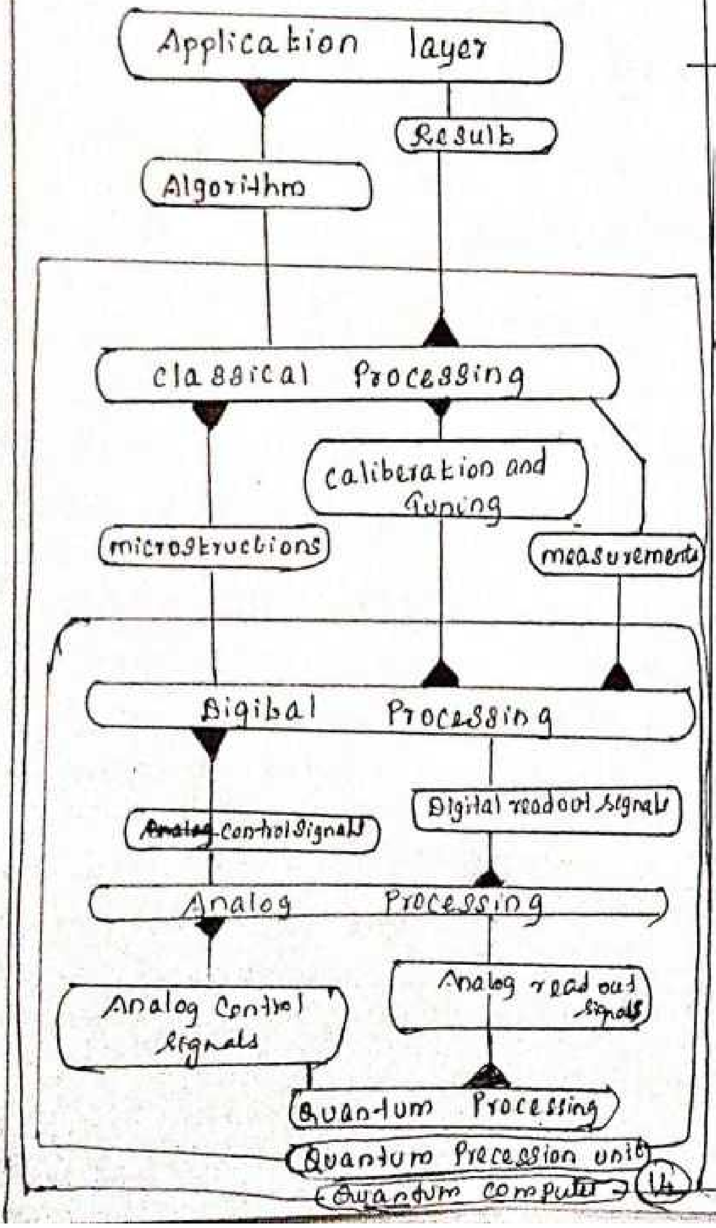
Application layer

- Not a part of quantum computer.
- used for representing user interface, operating System for a quantum Computer, coding environment.

- Formulating suitable quantum algorithms
- Hardware independent

Classical layer

- It optimizes and compiles the quantum algorithm into micro instructions.
- It also processes quantum-state measurement returned back from hardware from the below layers



Digital layer

- It interprets micro instructions into signals needed by qubit which act as quantum logic gates.
- Required analog pulses in the below layers.
- Feedback to the above classical layer for merging the quantum outcomes to the final results.

Analog layer.

- Creates voltage signals
- Phase and amplitude modulations.

Quantum layer

- It is integrated with the digital and the analog processing layer onto the same chip.
- Error correction is handled.
- Quantum processing unit (QPU) made up of three layers

i) Digital processing layer
ii) Analog processing layer
iii) quantum processing layer.

- Digital and analog operates at room temperature

⑦ Explain quantum States

- The fundamental law of quantum mechanics gives the idea of wave - particle duality.
- described by a complex function ψ
- depends on the coordinate x and time t

$$\boxed{\text{quantum State} \sim \psi(x,t)}$$

- Theory can only Predict the probability of the outcome of an experiment.
- $|\psi(x,t)|^2$ is a Probability per unit length or probability density.
- Along the real axis must be unity

$$|\psi|^2 = \int |\psi(x,t)|^2 dx = 1$$

Hilbert Space is defined as an infinite - dimensional vector Space with an inner product and is associated norm.

The state is represented as $|\psi_n\rangle$ is given by

$$|\psi_a\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \cdots - \alpha_i|i\rangle + \cdots + \alpha_{n-1}|n-1\rangle$$

The product of the bra vector $\langle\psi_a|$ and the ket vector $|\psi_a\rangle$ is assumed to be normalized. i.e) $\langle\psi_a|\psi_a\rangle = 1$.

$$\alpha_0^2 + \alpha_1^2 + \cdots \alpha_{n-1}^2 = 1$$

$$\sum_{i=0}^{n-1} |\alpha_i|^2 = 1$$

where

$$|0\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad |1\rangle \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \quad |2\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \quad |3\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

- In Hilbert Space H_2, two different bases $\{|0\rangle, |1\rangle\}$
- Superposition state is represented as

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \text{ and } |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

⑧ Define a) classical bits b) quantum bit ~~c) CNOT~~

c) Multiple qubits.

a) Classical Bits

Classical bit is an abstraction of a physical system, in any one of two states either '0' or '1'. Hence it can take the value 0 or 1. The bit is a smaller and simpler physical system.

- bits are stored on a silicon chip or a metal hard drive platter or on a magnetic tape.
- A bit can store one piece of information.
- Large amounts of information can be stored in a list of bits.
- Classical bit act as 1 or 0 state.

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b) Quantum bits

In quantum computing, a qubit or quantum bit is the basic unit of quantum information. The qubit is the quantum version of classical binary bit physically realized with a two state device. It is a two state quantum mechanical system showing the pecularity of quantum mechanics.

- quantum bit is a two level quantum system.
- described by a two dimensional complex Hilbert space.
- Orthogonal quantum states

$$|0\rangle \equiv \begin{bmatrix}1\\0\end{bmatrix},\ |1\rangle \equiv \begin{bmatrix}0\\1\end{bmatrix}$$

- Two states forms a computational basis.
- From the superposition principle

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where the amplitudes α and β complex numbers

- normalization condition

$$|\alpha|^2 + |\beta|^2 = 1$$

The generic state of a qubit be written as

$$|\varphi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix}$$

$$\left(0 \le \theta \le \pi, 0 \le \varphi < 2\pi\right)$$

- a continuum of states is allowed
- only single bit of information ($\sigma_n = +1$, or $\sigma_n = -1$)
- single qubit states are required to obtain α and β.

As the probability

$$|\alpha|^2 + |\beta|^2 = 1$$

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c) Multiple Qubits

- consider a system of two qubits in a four dimensional vector space.
- Hilbert space, four computational basis states as $|00\rangle, |01\rangle, |10\rangle, |11\rangle$
- Linear combination of the basis vectors

$$|\varphi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11}$ are complex coefficients.

- The probabilities of the four states are $|\alpha_{00}|^2, |\alpha_{01}|^2, |\alpha_{10}|^2, |\alpha_{11}|^2$
- Sum of the probabilities of the states

$$|\alpha_{00}^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

The probability for the first qbit to 0

$$P_0' = |\alpha_{00}|^2 + |\alpha_{01}|^2$$

Probability for 1 is

$$P_1' = |\alpha_{10}|^2 + |\alpha_{11}|^2$$

Sum of two probabilities is unity (1)

$$P_0' + P_1' = 1$$

after the measurement first qubits are 0 and 1, the state is

$$|\psi_0'\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$$

and

$$|\psi_1'\rangle = \frac{\alpha_{10}|0\rangle + \alpha_{11}|11\rangle}{\sqrt{|\alpha_{10}|^2 + |\alpha_{11}|^2}}$$

For the measurement of second qbit only.

The Probability for second qbit to 0 and 1

$$P_0'' = |\alpha_{00}|^2 + |\alpha_{10}|^2$$

$$P_1'' = |\alpha_{01}|^2 + |\alpha_{11}|^2$$

The corresponding states after measurements are

$$|\psi_0''\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{10}|10\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{10}|^2}}$$

and

$$|\psi_1''\rangle = \frac{\alpha_{01}|01\rangle + \alpha_{11}|11\rangle}{\sqrt{|\alpha_{01}|^2 + |\alpha_{11}|^2}}$$

Let us consider a special state of two qbits system

$$\alpha_{00} = \alpha_{11} = \frac{1}{\sqrt{2}}$$

$$\alpha_{01} = \alpha_{10} = 0.$$

This state is called Bell state and this pair of qbits is called EPR (Einstein, Podolsky and Rosen) pair.

when the two qbit system is in Bell state, the Probability of first qbit as 0 is 1/2 and that of 1 is 1/2

After measurement states

$$[\psi'_0] = |00\rangle$$

$$\therefore |\psi'_0\rangle = \frac{\alpha_{00}|00\rangle + \alpha_{01}|10\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$$

$\alpha_{00} = \frac{1}{\sqrt{2}}$ and $\alpha_{10} = 0$

$$|\varphi'_0\rangle = \frac{\frac{1}{\sqrt{2}}|00\rangle + 0|10\rangle}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2}}$$

$$= \frac{\frac{1}{2}|00\rangle}{\frac{1}{\sqrt{2}}}$$

$\varphi'_0 = |00\rangle$

and

$|\varphi'_1\rangle = |11\rangle$

Similarly for second qbit

$|\varphi'_1\rangle = |00\rangle$

$\alpha_{00} = \frac{1}{\sqrt{2}}$ and $\alpha_{10} : 0$

$$|\varphi'_0\rangle = \frac{\frac{1}{\sqrt{2}}|00\rangle + 0|10\rangle}{\sqrt{\left(1/\sqrt{2}\right)^2 + 0^2}}$$

$$= \frac{\frac{1}{\sqrt{2}}|00\rangle}{1/\sqrt{2}}$$

$\varphi'_0 = |00\rangle$ and

$|\varphi''_1\rangle = |11\rangle$

There are four special states called Bell states. These form an orthonormal basis as

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

and

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

⑨ Explain about Bloch Sphere

- understanding the qubits.
- It provides a geometric picture of the qubit and of the transformations takes on the state of a qubit.
- The normalization condition of states results that the qubit's state can be represented by a point on a sphere of unit radius called the Bloch sphere.
- The basis quantum states and the superposition states of the basis states are represented on Bloch sphere

The superposition state of a qubit is represented as

$$|\psi\rangle = e^{i\gamma}\left[\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right] \longrightarrow (1)$$

- real numbers θ, φ and γ

$$|\psi\rangle = e^{i\gamma}\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$= \alpha_0|0\rangle + \alpha_1|1\rangle \longrightarrow (2)$$

where $\alpha_0 = e^{i\gamma}\cos\frac{\theta}{2}$

$$\alpha_1 = e^{i\gamma}e^{i\varphi}\sin\frac{\theta}{2}$$

Eqn (1) becomes

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \longrightarrow (3)$$

The values θ and φ represent a point on the unit three dimensional sphere. This sphere is called Bloch sphere

$$|\psi\rangle = \frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$$

$$\alpha_0 = \alpha_1 = \frac{1}{\sqrt{2}}$$

Quantum state

$$\alpha_0 = \cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}, \quad \frac{\theta}{2} = 45^\circ$$

$$\text{or } \theta = 90^\circ$$

$$\alpha_1 = e^{i\varphi}\sin\frac{\theta}{2} = \frac{1}{\sqrt{2}} \Rightarrow e^{i\varphi} = 1$$

$$\varphi = 0$$

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

Any state $|\psi\rangle$ can be represented in terms of ket vectors $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

with $0 \le \theta \le \pi$ and $0 \le \varphi < 2\pi$

(10) Advantages and Disadvantages of Quantum computing over classical Computing

Advantages

1. quantum computers can Solve the complex mathematical problems.
   - Traditional computers find impossible to solve in a practical time frame.
2. The computing power is sufficient to process excessively large amounts of data.
3. Due to the teleporation phenomenon known as 'quantum tunneling', it can work in parallel. It uses less amount of electricity
   - reducing the power consumption upto 100 to 1000 times.
4. A computer is "thousands of times" faster than any classical computer.
5. can work without being overheated.
6. It can easily solve optimization problems.

Disadvantages

1. Due to advancements in quantum computers, the Security of the existing Internet of Things (IOT) would fall down.
2. Quantum computers will work as a different device and cannot replace classical computers.
3. It has not been invented completely only parts are being implemented.
4. It leads to 'DeCoherence' which is a loss of coherence in quantum.
5. Quantum Processors are very unstable and are very hard to test even.

⑪ Write short notes on i) one-Qubit quantum States
ii) Two Qubit quantum States - CNOT Gate

i) one-qubit quantum states

A one-qubit gate transforms an input qubit $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ into an output qubit $|\varphi\rangle = \alpha_0'|0\rangle + \alpha_1'|1\rangle$

a gate G is represented by a 2×2 transfer matrix with complex entries

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$$

The normalization conditions are $|\alpha_0|^2 + |\alpha_1|^2 = 1$ and $|\alpha_0'|^2 + |\alpha_1'|^2 = 1$

G must be a unitary matrix.

For a single qubit gate the eqn can be written as

$$\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_0' \\ \alpha_1' \end{pmatrix}$$

Thus

$\alpha_0' = g_{11}\alpha_0 + g_{12}\alpha_1$ and

$\alpha_1' = g_{21}\alpha_0 + g_{22}\alpha_1$

A few important one-qubit gates and their transfer matrices

1. $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ I identity gate - it leaves a qubit unchanged.
2. $X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the X or Not gate - it transpose the components of a qubit.
3. $Y = \sigma_x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ the Y gate - it multiplies the input qubit by i and flips the two components.
4. $Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ the z-gate - it changes the phase of a qubit.
5. $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ the Hadamard gate H - it creates a super position state from pure input states.

I is identity matrix

$X = \sigma_x$, $Y = \sigma_y$, $Z = \sigma_z$ are Pauli matrices.

The output vectors of these gates $|\varphi\rangle$, for a given input $|\psi\rangle = \alpha_0|0\rangle$ are given below

$$I|\psi\rangle = |\varphi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

$$|\varphi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

$$|\varphi\rangle = \sigma_x|\psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_0 \end{pmatrix}$$

or $|\varphi\rangle = \alpha_1|0\rangle + \alpha_0|1\rangle$

$$|\varphi\rangle = \sigma_y|\psi\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = i\begin{pmatrix} -\alpha_1 \\ \alpha_0 \end{pmatrix}$$

or $|\varphi\rangle = -i\alpha_1|0\rangle + i\alpha_0|1\rangle$

$$|\varphi\rangle = \sigma_z|\psi\rangle = \begin{pmatrix} 1 & - \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ -\alpha_1 \end{pmatrix}$$

or $|\varphi\rangle = \alpha_0|0\rangle - \alpha_1|1\rangle$

$$|\varphi\rangle = H|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

or $|\varphi\rangle = \frac{\alpha_0}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + \frac{\alpha_1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

The Hadamard gate H, when applied to a purse state $|0\rangle$ or $|1\rangle$, creates a superposition state

$$H|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

and

$$H|1\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

In general, the transformation of a qubit $|x\rangle$, with $x = 0$ or $x = 1$, carried out by a Hadamard gate can be expressed as

$$|x\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^x|1\rangle\right)$$

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ii) Two-Qubit Quantum Gates - CNOT Gate.

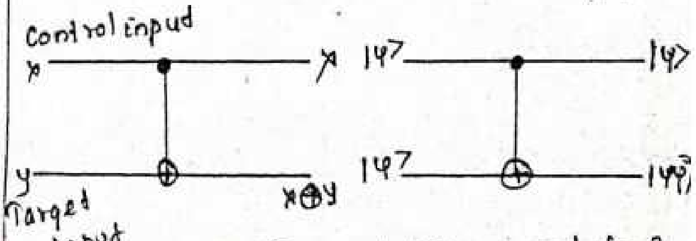
- A gate with two inputs and two outputs is called CNOT.
- The first output is called the control and the second is called the target.

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle,$$

$$|\varphi\rangle = \beta_0|0\rangle + \beta_1|1\rangle$$

The input vector of the quantum CNOT gate is

$$|V_{CNOT}\rangle = |\psi\rangle \otimes |\varphi\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \otimes \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_0 \\ \alpha_1\beta_1 \end{pmatrix}$$



$\oplus \rightarrow$ addition modulo 2

The components of the input vector are transformed by the CNOT quantum gate is follows

$|00\rangle \rightarrow |00\rangle \quad |01\rangle \rightarrow |01\rangle \quad |0\rangle \rightarrow$

$|11\rangle \quad |11\rangle \rightarrow |10\rangle$

first qubit is control input and second qubit is target input.

The transfer matrix G_{CNOT} of the CNOT quantum gate can be given as a sum of the outer products of the components of the output and input vectors.

$$G_{CNOT} = |00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$$

The outer products of the basis vectors $|00\rangle$ and $|01\rangle$ with themselves as well as the outer products of $|10\rangle$ with $|11\rangle$ and $|11\rangle$ and $|10\rangle$

$$|00\rangle\langle 00| = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} (1000) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|01\rangle\langle 01| = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} (0100) =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|10\rangle\langle 11| = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} (0001) =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|11\rangle\langle 10| = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} (0010) =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The transition matrix of the circuit is

$$G_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

It is easy to determine the output state vector $|W_{CNOT}\rangle$ given the input state vector $|V_{CNOT}\rangle$ and the transfer matrix of a CNOT gate

$\alpha_{00} = \frac{1}{\sqrt{2}}$ and $\alpha_{10} = 0$

$$|\psi'_0\rangle = \frac{\frac{1}{\sqrt{2}}|00\rangle + 0|10\rangle}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2}}$$

$$= \frac{\frac{1}{2}|00\rangle}{\frac{1}{\sqrt{2}}}$$

$\psi'_0 = |00\rangle$

and

$|\psi'_1\rangle = |11\rangle$

Similarly for second qbit

$|\psi''_0\rangle = |00\rangle$

$\alpha_{00} = \frac{1}{\sqrt{2}}$ and $\alpha_{10} = 0$

$$|\psi'_0\rangle = \frac{\frac{1}{\sqrt{2}}|00\rangle + 0|10\rangle}{\sqrt{\left(1/\sqrt{2}\right)^2 + 0^2}}$$

$$= \frac{\frac{1}{\sqrt{2}}|00\rangle}{1/\sqrt{2}}$$

$\psi'_0 = |00\rangle$ and

$|\psi''_1\rangle = |11\rangle$

There are four special states called Bell states. These form an orthonormal basis as

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

and

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$